TEST 1R

QUESTION 7 [Marks 10]

Three vectors are given by

$$\mathbf{r} = 6\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$$
$$\mathbf{s} = -2\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$$
$$\mathbf{t} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

(a) (i) Determine the vector $\mathbf{a} = \mathbf{s} + \mathbf{t}$.

(ii) Determine the scalar $b = \mathbf{r} \cdot (\mathbf{s} + \mathbf{t})$.

(iii) Determine the vector $\mathbf{c} = \mathbf{s} \times \mathbf{t}$.

(b) A sniper fires a bullet at a car as it is travelling north at 120 km/h. The bullet speed is 400 m s^{-1}. The bullet enters the car at a point exactly at the centre of the forward side window on the east of the car. The bullet passes through the window with its velocity unchanged. The bullet crosses the car parallel to the windscreen and exits at the centre of the side window that is directly opposite the window through which it entered.

(i) Draw a vector diagram showing the velocity of the bullet $\mathbf{v}_b$, the velocity of the car $\mathbf{v}_c$, and the velocity of the bullet relative to the car $\mathbf{v}_r$.

(ii) Determine the direction the bullet is travelling.

QUESTION 8 [Marks 12]

A lift of mass of 2100 kg is carrying a person of mass 50kg. The lift is supported by a cable hanging vertically. The lift is initially moving down at a speed of 15 ms^{-1} and it is brought to rest with a constant acceleration over a distance of 15 m.

(a) With the help of a sketch, identify and determine all the external forces (magnitude and direction) acting on the lift during the acceleration.

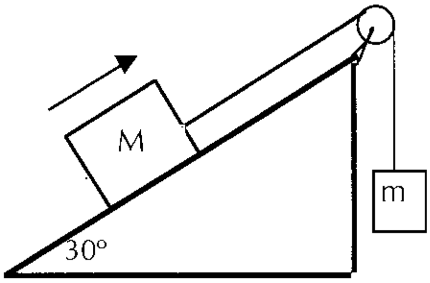
(b) A mass of $M = 0.17$ kg is moving on a frictionless horizontal surface at a speed of 1.6 ms^{-1}. It collides with a mass m which is initially at rest. The collision is elastic. After the collision the first mass continues in its original direction with a speed 0.7 ms^{-1}.

 (i) Determine the mass m.

 (ii) Determine the speed of the mass m after the collision.

QUESTION 9 [Marks 12]

A mass of $M = 7.0$ kg moves along a rough plane that is inclined at an angle of 30° to the horizontal. It is connected by a light string over a frictionless pulley to a block of mass $m = 9.0$ kg that hangs vertically. The mass m moves down with constant velocity



(a) With the help of a sketch,

 (i) identify all the forces acting on body M

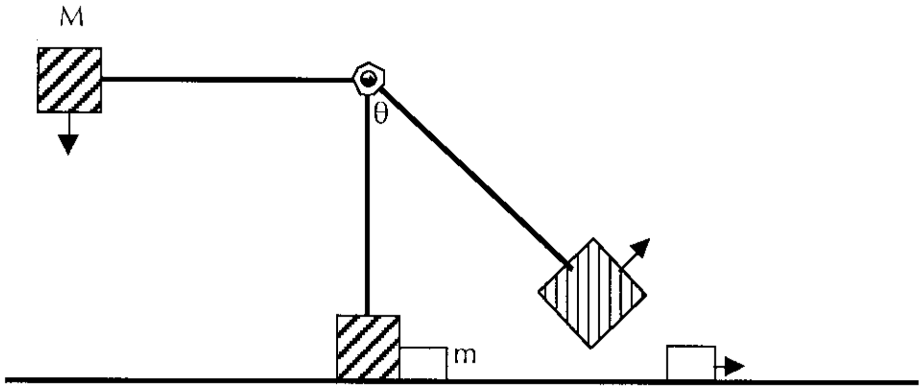
 (ii) identify all the forces acting on body m.

(b) Determine the tension in the string.

(c) Determine the frictional force (magnitude and direction) acting on mass M.

(d) Determine the coefficient of kinetic friction μ_k acting between the mass M and the plane.

QUESTION 10 [Marks 16]



A mass of $M = 0.25$ kg is attached to a massless rod 0.5 m long that is fixed at its opposite end. Initially the object is held so the rod is taut and is horizontal. At the bottom of its path the mass M inelastically strikes a second mass of $m = 0.1$ kg which is resting on a rough horizontal plane with coefficient of kinetic friction $\mu_k = 0.25$. Immediately after the collision the mass $M = 0.25$ kg is moving to the right with velocity $v = 1.37 \text{ m s}^{-1}$.

(a) Determine the velocity of the mass $M = 0.25$ kg immediately before the collision.

(b) Determine the velocity of the mass $m = 0.1$ kg immediately after the collision. Establish that mass m moves off faster than the mass M.

(c) Determine the angle θ through which the $M = 0.25$ kg mass swings before stopping.

(d) Identify in a diagram all the forces acting on the $m = 0.1$ kg mass after the collision.

(e) Determine the horizontal distance the $m = 0.1$ kg mass moves before coming to rest.

QUESTION 11 [Marks 12]

A merry-go-round is a uniform circular disc of radius 0.8 m and mass 90 kg that can rotate about an axis perpendicular to its centre. The merry-go-round is accelerated in a time of 4 s at a constant rate from rest to a rotation speed of one revolution every 2.5 s. It then rotates freely on frictionless bearings. The moment of inertia of a uniform disc of mass M and radius R is $I = \frac{1}{2}MR^2$.

(a) (i) Determine the angular acceleration of the merry-go-round.

(ii) Determine the torque that accelerates the merry-go-round.

(iii) Determine the work done by the torque while it accelerates the merry-go-round.

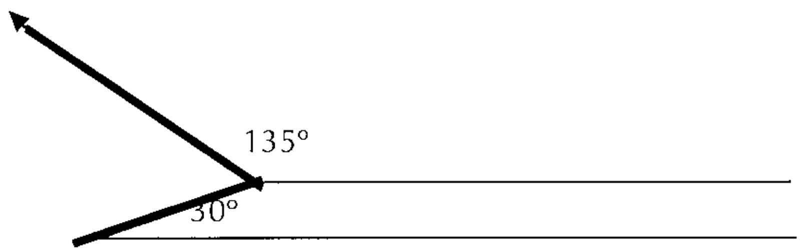
(b) While the merry-go-round is rotating freely a child of mass 30 kg jumps vertically down onto the merry-go-round near its outside edge. Determine the new angular speed of the merry-go-round with the child standing on it.

(c) While the merry-go-round continues to rotate freely, the child walks to the central rotation axis. Determine the new angular speed of the merry-go-round with the child standing at its axis.

QUESTION 12 [Marks 8]

(a) A metre rule balances horizontally on a knife-edge at the 500 mm mark. Two identical coins are placed on the ruler, one at the 80 mm mark and the other at the 140 mm mark. Each coin has a mass of 8 g. The rule is now found to balance on the knife-edge at the 420 mm mark. Determine the mass of the metre rule.

(b)



A cricket ball **of mass 150 g** approaches a bat in a direction 30° above the horizontal at a speed of 66 ms^{-1}. After colliding with the bat the ball moves off at an angle of 135° above the horizontal as shown at a speed of 90 m s^{-1}. The collision lasts 0.7 ms.

(i) Determine the average force $\bar{F} = \int dt F(t)$ exerted by the bat on the ball.

(ii) Determine the impulse **I** of the force.

QUESTION 13 **[Marks 10]**

An object is thrown upwards from the earth's surface at a velocity of $v = 2\sqrt{gR_e}$, where R_e is the radius of the earth.

(a) Calculate its velocity when the object is a distance of $5R_e$ above the earth's surface.

(b) Show that the object will escape from the earth, and express its final velocity very far from the earth in terms of g and R_e.