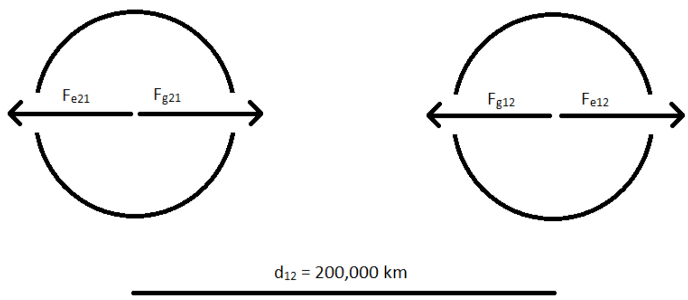
Question 1: 12 marks, 16 for higher.

A highly advanced alien race decides they want to play pool with planets. In the vast void between galaxies, they take two Earth-mass planets (coated with a perfectly conducting metal), and place them so that their centres are 200,000 km apart. In order to balance the gravitational attraction between the planets, the aliens add equal numbers of electrons to each planet so that the electric force between the planets precisely balances the attraction between them due to gravity.

a) Draw a figure showing the forces acting on each of the planets



Forces on planet 1
Forces on planet 2
Gravitational forces attract, Electric forces repel

b) How many electrons do the aliens have to add to each planet such that the electric force exactly balances the gravitational force between the planets?

Electric force plus gravitational force match. Thus $F_{e12} = -F_{g12}$

Therefore: $\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{G m_1 m_2}{r^2}$

$m_1 = m_2$ and $q_1 = q_2$, rearranging to get q_1

$q_1 = \sqrt{4\pi\varepsilon_0 G m_1^2}$

Sub numbers in

→ $q_1 = \sqrt{4\pi \times 8.854\times10^{-12}\ \mathrm{F.m^{-1}} \times 6.673\times10^{-11}\mathrm{m^3kg^{-1}s^{-2}} \times (5.97\times10^{24}\ \mathrm{kg})^2}$

Thus $q_1 = 5.144\times10^{14}$ C

Total number of electrons = Total Charge / Charge on 1 electron

$n = \frac{q_1}{q_e} = \frac{5.144\times10^{14}}{1.602\times10^{-19}} = 3.211\times10^{33}$

c) What mass of electrons does this correspond to?

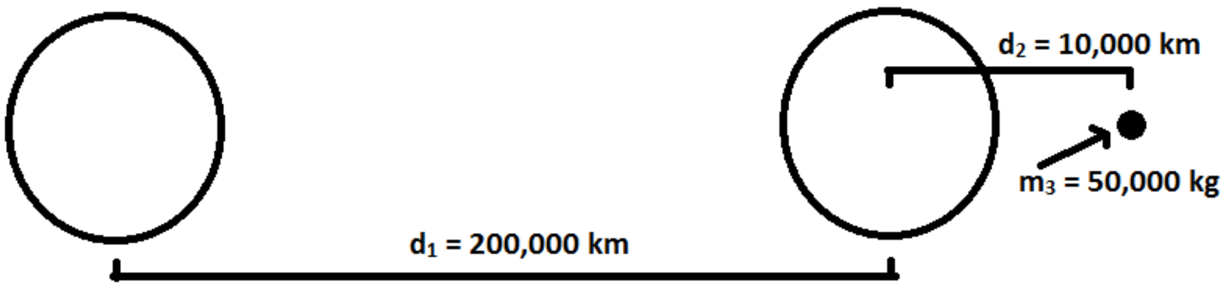
Total mass = Number of electrons x mass of an electron

Total mass = $3.211 \times 10^{33} \times 9.109 \times 10^{-31}$ kg $= 2.925 \times 10^{3}$ kg $= 2925$ kg

d) Assuming the planets are perfectly spherical, how are the electrons distributed once added?

As the planets are covered in a perfectly conducting metal, the electrons spread out evenly across the surfaces of the planets.

e) HIGHER ONLY: Alien saboteurs decide to disrupt the game, and place a 50,000 kg ball of protons 10,000 km away from one of the planets, on the opposite side of that planet to the other. Describe the series of events that occurs until the system of the ball and the planets comes to a rest. You can assume that any collisions are perfectly inelastic (i.e. sticky)



The ball of protons will be strongly attracted to the planet it is placed above, and so the two will accelerate towards each other.

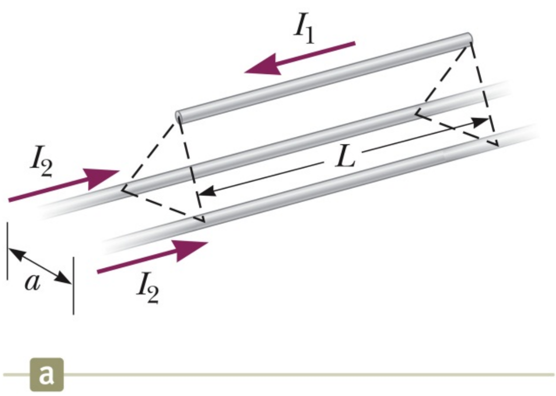
The force between the planet and the ball of protons will be roughly double the force due to gravity.

When the ball and planet collide, the protons will spread evenly across the surface. The net charge of the planet will now be smaller (by about 1%).

Because the planet no longer has enough negative charge for the electric force to balance its attraction to the other planet, their attraction due to gravity will now outweigh their electrical repulsion, and they will accelerate towards each other.

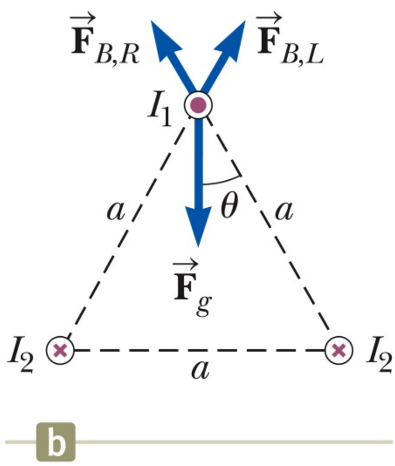
Finally, the planets will collide, and will be left as one object, with the remaining charge evenly distributed across its surface.

Question 2: 10 marks



Two infinitely long, parallel wires are lying on the ground a distance a = 12.5 cm apart. A third wire, of length L = 100.0 m, and mass 35.0 kg, carries a current of I_1 = 1000 A, and is levitated above the first two wires at a horizontal position midway between them, as can be seen in the figure. The two parallel wires carry equal currents (I_2) in the opposite direction to that flowing through the levitated wire.

a) Draw a figure showing the situation as viewed end on. Show the various forces acting on the levitated wire, and the currents flowing through the wires.



b) What current must the infinitely long wires carry so that the three wires form an equilateral triangle?

Vertical forces have to cancel for wire to be in equilibrium. In other words, there is no net vertical force on the wire, i.e. $F_{B\_vertical} + F_g = 0$

$F_{B\_vertical} = \frac{\mu_0 I_1 I_2 l}{2\pi a}\cos\theta$ for each of the two wires

So total F_B is: $F_{B\_vertical} = \frac{\mu_0 I_1 I_2 l}{\pi a}\cos\theta$

The force due to gravity, F_g is: $F_g = -mg$

Therefore: $\frac{\mu_0 I_1 I_2 l}{\pi a}\cos\theta + (-mg) = 0$

Then: $I_2 = \frac{mg\pi a}{\mu_0 I_1 l\cos\theta}$

Substitute numbers: $I_2 = \frac{35\text{ kg} \times 9.80\text{ m.s}^{-2} \times 3.14 \times 0.125\text{ m}}{4 \times 3.14 \times 10^{-7}\text{ T.m.A}^{-1} \times 1000\text{ A} \times 100\text{ m} \times \cos 30^{\circ}}$

$I_2 = 1238\text{ A} = 1240\text{ A}$

Question 3: **12 marks, 16 for higher.**

You have two identical capacitors, each of which has a capacitance of 10,000 F, and a 120 V battery.

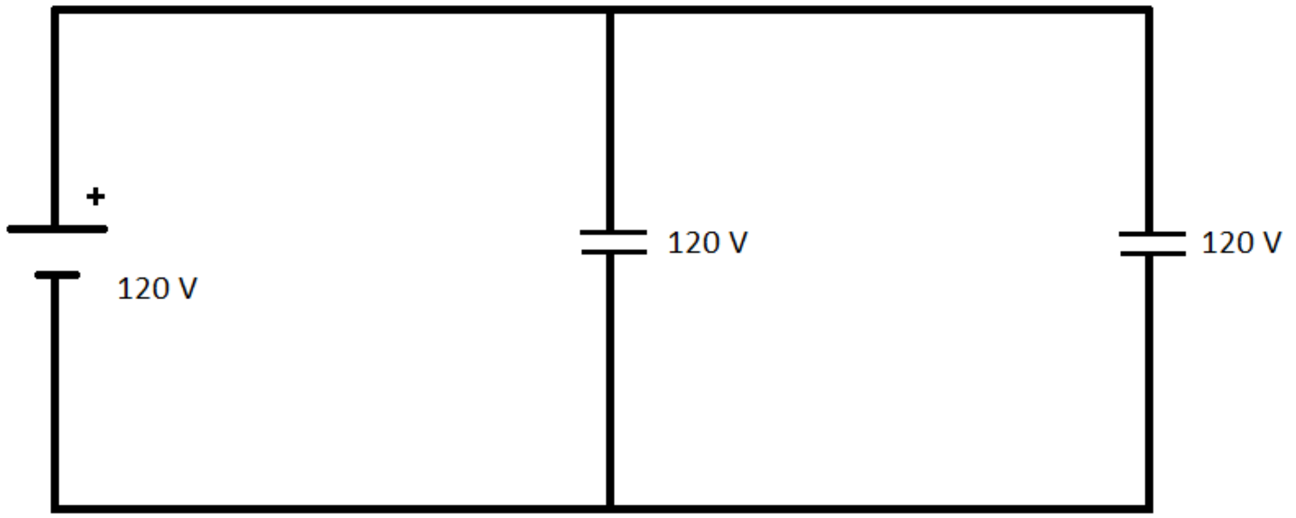
a) Calculate the total energy stored in both capacitors if they are connected *in series* across the battery. Draw a circuit diagram showing how the capacitors are connected to the battery in this case.

: For the figure

$E = 2\times\frac{1}{2}CV^2$ (2 since there are 2 capacitors)

$E = 2\times\frac{1}{2}\times 10000\text{ F}\times(60\text{ V})^2 = 3.6\times 10^7\text{ J}$

b) Calculate the total energy stored in both capacitors if they are connected in *parallel* across the battery. Again, draw a circuit diagram showing how the capacitors are connected to the battery.



: For the figure

Again, $E = 2\times\frac{1}{2}CV^2$

$E = 2\times\frac{1}{2}\times 10000\text{ F}\times(120\text{ V})^2 = 1.4\times 10^8\text{ J}$

A thunderstorm is essentially a giant parallel-plate capacitor. Imagine a giant thunderstorm over Sydney. For simplicity, assume that the base of the thundercloud is circular, with a radius of 15.0 km, and that the cloud base is 1 km above the ground.

c) Calculate the capacitance of the thunderstorm.

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0(\pi r^2)}{d} = \frac{8.854\times10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}} \times (3.142 \times (15000\ \mathrm{m})^2)}{1000\ \mathrm{m}}$$

$$C = 6.26\times10^{-6}\ \mathrm{F} = 6.26\ \mu\mathrm{F}$$

A lightning strike causes the thunderstorm to completely discharge, transferring 2.53 C of charge and 1.94 x 10^6 J of energy to the Earth.

d) Calculate the potential difference that existed between the cloud and the ground just prior to the lightning flash.

$$\Delta V = \frac{2\Delta U}{q}$$

$$\Delta V = \frac{2 \times 1.94 \times 10^6\ \mathrm{J}}{2.53\ \mathrm{C}} = 1.53 \times 10^6\ \mathrm{V}$$

e) HIGHER ONLY As the air between the cloud and the ground becomes more humid, what happens to the capacitance of the thunderstorm? Will the maximum amount of energy the storm can store before a lightning strike increase, or decrease? Explain your answer. Assume that the maximum potential difference the storm can store remains the same.

As the air gets more humid, the dielectric constant of the capacitor increases.
The capacitance therefore also increases ($C = \kappa C_0$)
The energy stored by a capacitor is proportional to the capacitance, *C*, and the potential difference, *V*.

Since the maximum potential difference is constant, and the capacitance increases, the maximum amount of energy the storm can store will also increase

Question 4: 18 marks.

a) Explain, in words, Gauss' law for electric flux.

The net flux through any closed surface surrounding a charge q is given by $\frac{q}{\varepsilon_0}$, and is independent of the shape of that surface

b) Give the mathematical form for Gauss' law for electric flux

$$\Phi_E = \oint \boldsymbol{E}.d\boldsymbol{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$

An insulating solid sphere of radius, a, has a uniform volume charge density, and carries a total positive charge of magnitude, Q.

c) Calculate the magnitude of the electric field at a point outside the sphere

Chose a spherical Gaussian surface of radius r, concentric with the sphere. Everywhere on that sphere, $\boldsymbol{E}.d\boldsymbol{A} = E\ dA$.

Using Gauss' law: $\Phi_E = \oint \boldsymbol{E}.d\boldsymbol{A} = \oint E\ dA = \frac{Q}{\varepsilon_0}$

Since E is constant everywhere on the sphere, we take it out of the integral, giving us $E \oint dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$

Therefore, for $r > a$, we have: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

d) Calculate the magnitude of the electric field at a point inside the sphere

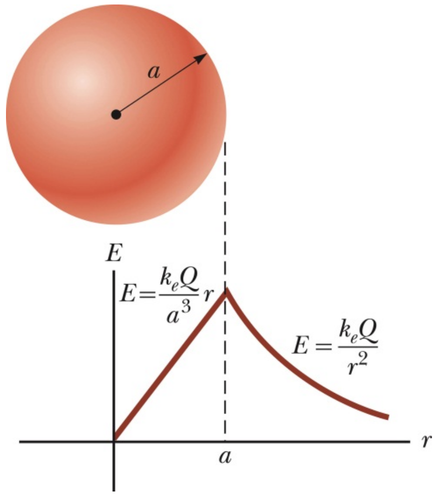
The charge, q, interior to radius r is: $q_{\text{in}} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right)$

Again, for a sphereical Gaussian surface, as before: $E(4\pi r^2) = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{4\pi r^3 \rho}{3\varepsilon_0}$

We know that $Q = \frac{4}{3}\pi\rho a^3$, therefore $\rho = \frac{3Q}{4\pi a^3}$

Therefore, for $r < a$, we have: $E = \frac{Q}{4\pi\varepsilon_0 a^3} r$

e) Draw a graph showing the variation of the electric field as a function of distance, *r*, from the centre of the sphere.



f) Give the mathematical form for Ampère's law, and define the terms used

$$\oint \boldsymbol{B}.\boldsymbol{ds} = \mu_0 I$$

The line integral of the magnetic field, ***B***, around a path ***ds*** equals $\square_0 I$, where *I* is the total steady current passing through any surface bounded by the closed path, and $\square_0$ is the permeability of free space.

A long, straight wire of radius R carries a steady current *I* that is uniformly distributed through the cross section of the wire.

g) Calculate the magnetic field a distance *r* from the centre of the wire in the region $r \geq R$

Chose a circular path for our integral a distance *r* from the wire's centre. On that path, ***B*** must have constant magnitude, and be parallel to ***ds*** at all points.

Applying Ampère's law, noting the total current passing through is *I*.

$$\oint \boldsymbol{B}.\boldsymbol{ds} = B \oint \boldsymbol{ds} = B(2\pi r) = \mu_0 I$$

$B = \frac{\mu_0 I}{2\pi r}$, for $r > R$

h) Calculate the magnetic field in the region $r < R$.

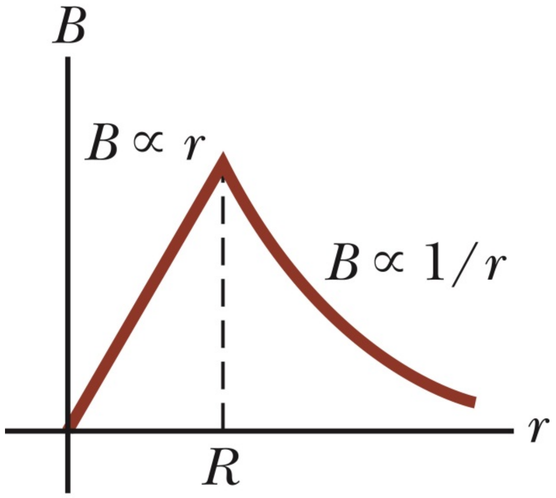
For *r* less than R, the current inside the loop will be given by $\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$

Therefore $I' = \frac{r^2}{R^2} I$

Applying Ampère's law, $\oint \boldsymbol{B}.\boldsymbol{ds} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I\right)$

Rearrange to get: $B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$ for $r < R$

i) Draw a graph showing the variation of the magnetic field as a function of the distance r from the centre of the wire



Question 5: **12 marks**

Hydrogen-alpha (H□) radiation is emitted when the electron in an excited hydrogen atom falls from the third to the second lowest energy level. When passing through a vacuum, H□ light has a wavelength of 656 nm, putting it in the red part of the visible electromagnetic spectrum.

a) What is the frequency of H□ radiation?

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\,\text{m s}^{-1}}{656\times10^{-9}\,\text{m}} = 4.57\times10^{14}\ \text{Hz}$$

An astronomer takes a photograph of the Orion nebula, a bright source of H□ light, using a lens made from fused quartz, which has a refractive index of 1.46 for red light.

b) What is the wavelength of the H□ radiation when it is passing through the lens?

The refractive index, *n*, is the ratio of the wave of light in vacuum to in material. I.e.

$$n = \frac{\lambda}{\lambda_{\text{material}}}.$$

So $$\lambda_{\text{material}} = \frac{\lambda}{n} = \frac{656\times10^{-9}\,\text{m}}{1.46} = 449\times10^{-9}\ \text{m} = 449\ \text{nm}$$

c) The aperture of the astronomer's lens is 3.00 cm. What is the limiting angle of its resolution for H□ light?

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D}$$

Therefore $$\theta_{\text{min}} = 1.22\frac{656\times10^{-9}\text{m}}{0.030\text{m}} = 2.67\times10^{-5}\ \text{rad} = 1.53\times10^{-3}\ ^{\circ} = 5.50\ \text{arcsec}$$

Hα is one of a series of spectral lines emitted by electrons falling to the second energy level in hydrogen. A second line, known as Hydrogen-beta (Hβ), falls at a wavelength of 486 nm, and is the result of electrons falling from the fourth to the second energy level.

Our astronomer wants to study the spectrum of the Orion nebula, and so uses a telescope to direct its light, at normal incidence, onto a diffraction grating with 600 lines/mm.

d) What is the angular separation between the first-order maxima for Hα and Hβ?

$$m\lambda = d\sin\theta$$

$$d = \frac{1\ \text{mm}}{600\ \text{lines}} = \frac{0.001\ \text{m}}{600} = 1.67\times10^{-6}\ \text{m}$$

So for Hα, with λ =656 nm, the first order maximum ($m = 1$) is at

$$\theta_{H\alpha} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{1\times656\times10^{-9}\ \text{m}}{1.67\times10^{-6}\ \text{m}}\right)$$

Therefore $\theta_{H\alpha} = 23.1°$

So for Hβ, with λ=486 nm, the first order maximum ($m = 1$) is at

$$\theta_{H\beta} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{1\times486\times10^{-9}\ \text{m}}{1.67\times10^{-6}\ \text{m}}\right)$$

Therefore $\theta_{H\beta} = 16.9°$

$$\theta_{\text{Sep}} = \theta_{H\alpha} - \theta_{H\beta}$$

Therefore $\theta_{\text{Sep}} = 23.1° - 16.9° = 6.2°$

e) The spectrum projected from the diffraction grating illuminates a photographic plate which can be moved towards and away from the grating. If the astronomer wants the two lines to be separated by 10 cm on the photographic plate, how far from the grating must the plate be placed?

$\sin\theta = \frac{\text{Separation}}{\text{Distance to screen}}$ or $\tan\theta = \frac{\text{Separation}}{\text{Distance to screen}}$

Therefore:

$D = \frac{S}{\sin\theta}$ or $D = \frac{S}{\tan\theta}$

$S = 0.10\ \text{m}$, $\theta = 6.2°$

$D = 0.93\ \text{m}$ or $D = 0.92\ \text{m}$

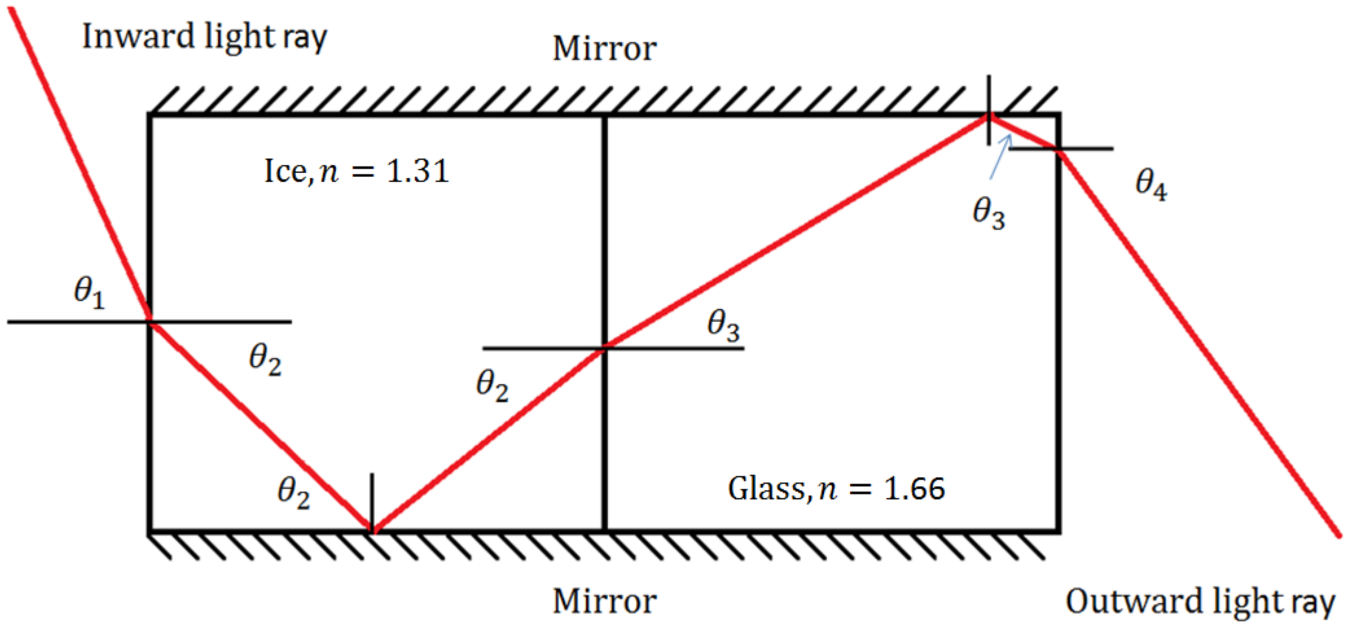
Question 6: **12 marks** **18 for Higher**

A student wants to experiment with the laws of reflection and refraction, and takes a cube of flint glass (with refractive index 1.66, and sides of length 10 cm) and an equal sized cube of pure ice (refractive index 1.31), and two plane mirrors, which are 20 cm long.

The student places the first mirror flat, face up, on the table, and places the two cubes on top of it, touching one another, and then places the second mirror, face down, on top of them.

The student then shines a ray of light at the sandwiched mirrors and cubes such that it hits the ice block halfway between the mirrors, incident at an angle of 75 degrees to the normal.

a) Draw a figure that shows the path of the light from the air, through the ice, then through the glass, and finally exiting from the glass.



: For showing the three refractions

: For showing the two reflections

b) How far from its entry point to the ice does the ray of light first hit a mirror, measured horizontally?

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = 47.5°$$

$$\tan\theta_2 = \frac{\text{Opposite}}{\text{Adjacent}} \quad \text{, Opposite} = 5 \text{ cm}$$

$$\text{Adjacent} = \frac{\text{Opposite}}{\tan\theta_2} = \frac{5 \text{ cm}}{\tan 47.5°}$$

$\text{Adjacent} = 4.58 \text{ cm}$ **[ALLOW FOR ROUNDING – i.e. 4.55 – 4.60 is OK]**

c) At what angle to the normal does the ray emerge from the ice into the flint glass?

$$n_2 \sin\theta_2 = n_3 \sin\theta_3$$

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3}\sin\theta_2\right) = 35.6°$$

d) At what angle to the normal does the ray emerge from the flint glass?

$$n_3 \sin\theta_3 = n_4 \sin\theta_4$$

$$\theta_4 = \sin^{-1}\left(\frac{n_3}{n_4}\sin\theta_3\right) = 75°$$

HIGHER ONLY

e) How far from the mirror lying on the table, vertically, does the ray of light leave the ice?

$\tan\theta_2 = \frac{\text{Opposite}}{\text{Adjacent}}$, Adjacent = 10 – 4.58 cm = 5.42 cm

$$\text{Opposite} = \text{Adjacent} \times \tan\theta_2 = 5.42\ \text{cm} \times \tan 47.5°$$

$$\text{Adjacent} = 5.91\ \text{cm}$$

[AGAIN, ALLOW FOR ROUNDING AND CARRY THROUGH ERROR HERE – i.e. 5.85 – 5.95 is OK]

HIGHER ONLY

f) How far from the mirror lying on the table, vertically, does the ray of light leave the glass?

For the triangle with the ray going from ice/glass interface to the mirror, vertical side is a total of 10 – 5.91 cm in length = 4.09 cm.

The horizontal side of that triangle is therefore:

$$\tan\theta_3 = \frac{\text{Vertical}}{\text{Horizontal}}$$

$$\text{Horizontal} = \frac{\text{Vertical}}{\tan\theta_3} = \frac{4.09\ \text{cm}}{\tan 35.6°}$$

$$\text{Horizontal} = 5.71\ \text{cm}$$

[AGAIN, ALLOW FOR ROUNDING AND CARRY THROUGH ERROR HERE – i.e. 5.65 – 5.75 is OK]

For the triangle going from the mirror to the glass/air interface, horizontal side is 10 – 5.71 cm in length = 4.29 cm.

The vertical side of that triangle is therefore

$$\text{Vertical} = \text{Horizontal} \times \tan\theta_3 = 4.29\text{ cm} \times \tan 35.6°$$

$$\text{Vertical} = 3.07\text{ cm}$$

So the distance from the mirror on the table is $\text{Distance} = 10\text{ cm} - 3.07\text{ cm} = 6.93\text{ cm}$

[AGAIN, ALLOW FOR ROUNDING AND CARRY THROUGH ERROR HERE – i.e. 6.85 – 7.00 is OK]

Question 7: 13 marks 17 for Higher

The peak intensity of the Solar spectrum falls in the yellow part of the spectrum, where the eye is most sensitive.

a) If that maximum intensity falls at a wavelength of 502 nm, calculate the temperature of the Sun's surface. State any assumptions you make.

Assume the Sun is a black body

$\lambda_{max}T = 2.898\times10^{-3}\text{m. K}$

Sub in numbers to get T = 5780 K

b) A leaf, which we can model as a rectangle with sides 5.0 cm and 2.0 cm long, is illuminated in full sunlight, and receives a flux of 1.0 W. Using the average distance from the Earth to the Sun, calculate the radius of the Sun. State your assumptions.

Area of the leaf is A = 0.05 x 0.02 = 0.001 m^2

Therefore, the flux per m^2 at the Earth from the Sun is $f = \frac{1\text{ W}}{0.001\, m^2} = 1000\text{ W m}^{-2}$

Therefore, the total power output of the Sun is

$P = 4\pi r_E^2 f = 4\pi\times(1.496\times10^{11})^2\times1000$

$P = 2.81\times10^{26}\ \text{ W}$

Stefan's Law states $P = \sigma A e T^4$

Assume the Sun is a black body (i.e. has emissivity = 1)

The Sun is a sphere, radius R, so area = $4\pi R^2$

Rearrange to get $R = \sqrt{\frac{P}{4\pi\sigma T^4}}$

Substitute number to get R

$R = 5.94\times10^8\text{ m} = 5.94\times10^5\text{ km}$

c) Is the radius you just calculated likely to be an overestimate, or an underestimate? State your reasoning.

Underestimate

The atmosphere absorbs some of the Sun's radiation

d) Higher only: Calculate how many photons per second are hitting the leaf. You can assume that all the photons have the same energy (i.e. that the Sun's light is monochromatic, at λ =502 nm).

Energy of 1 photon → $E = hf = \frac{hc}{\lambda}$

Thus $E = 6.63\times10^{-34}\text{ J. s}\times\frac{2.998\times10^8\text{ ms}^{-1}}{502\times10^{-9}\text{ m}} = 3.959\times10^{-19}\text{ J}$

Number of photons per second, $N = \frac{F}{E}$ (where F is the flux, and E is energy per photon)

Therefore $N = \frac{1}{3.959\times10^{-19}} = 2.526\times10^{18}$

Question 8: 11 marks [Total 100], 13 for higher [Total 120]

a) State the time-independent Schrödinger equation, defining each term that you use.

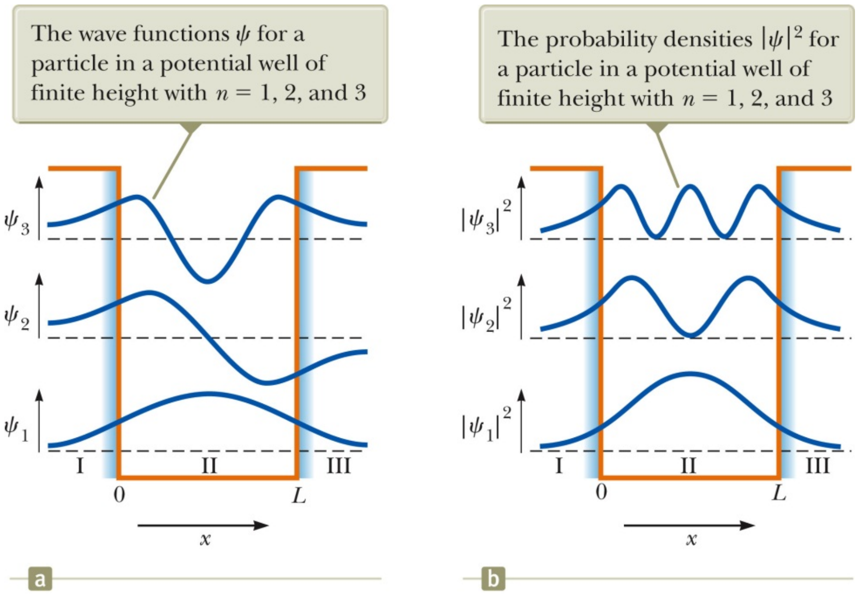
For a particle of mass *m*,

$$E\Psi = -\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + U\Psi$$

E is the total energy of the state □

U is the potential energy function

b) Sketch the wave functions, □, and probability densities $|\Psi^2|$ for a particle in a potential well with finite height, for n = 1, 2 and 3



c) Explain in words, and give the mathematical form of the Heisenberg uncertainty principle

If we simultaneously measure the position, *x*, and momentum *p* of a particle then the product of the uncertainties, □*x* and □*p* can never be less than □/2

$\Delta x \Delta p \geq \frac{\hbar}{2}$

d) HIGHER ONLY. Usain Bolt has a top speed of 44.7 km/h, and weighs 94 kg. Calculate the de Broglie wavelength of Usain Bolt when he is running at top speed.

$$\lambda = \frac{h}{mu}$$

44.7 km/h = 44700 m/h = 44700/3600 m/s = 12.42 m/s

$$\lambda = \frac{6.63\times10^{-34}\ \text{J.s}}{94\ \text{kg}\times12.42\ \text{ms}^{-1}} = 5.68\times10^{-37}\text{m}$$