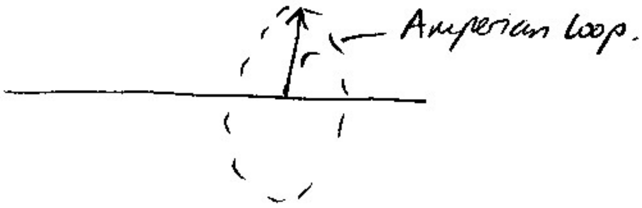
PHYS1231 Exam Solutions, Summer Session 08/09

Q1) a) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I_{enc}$.

The line integral of the magnetic field about any closed loop, $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l}$, is equal to μ_0 times the current enclosed by the loop, I_{enc}.

b) Need to chose the loop such that the magnetic field is constant around the loop $\rightarrow$ $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} \Rightarrow \underset{\sim}{B} \cdot \oint d\underset{\sim}{l}$, allowing us to determine B.

Here, circular loops concentric with the centre of the cable.



c). There is a uniform current density, I, distributed over the area of the wire, $J_1 = I/\pi a^2$ for the inner conductor.

i) $r < a$, $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I_{enc}$.

$$B \cdot 2\pi r = \mu_0 J_1 \cdot \pi r^2 \Rightarrow B(r) = \frac{\mu_0 I r}{2\pi a^2}.$$

ii). $a < r < b$, $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I_{enc}$

$$B \cdot 2\pi r = \mu_0 I \quad \Rightarrow \quad B(r) = \frac{\mu_0 I}{2\pi r}.$$

Now, the current density within the outer conductor is

$$\Rightarrow \quad J_2 = \frac{I}{\pi(c^2 - b^2)}$$, directed opposite to that of the first conductor.

iii) $b < r < c$, $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I_{enc}$

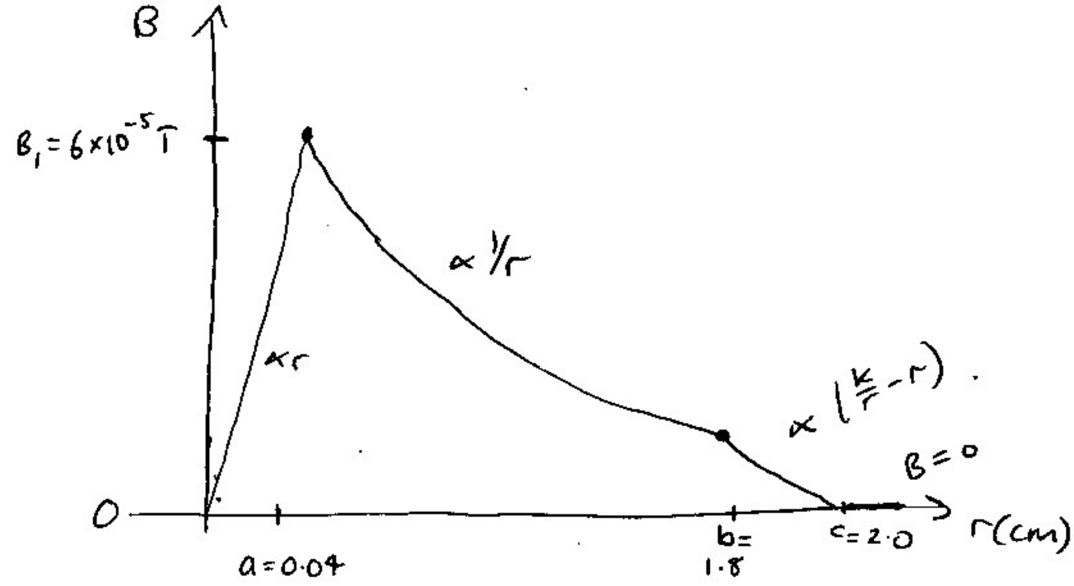
$$B \cdot 2\pi r = \mu_0 \left(I - \underset{(\pi(r^2 - b^2))}{\cancel{}} \cdot J_2\right) \Rightarrow B = \frac{\mu_0 I}{2\pi r} \cdot \frac{(c^2 - r^2)}{(c^2 - b^2)}$$

iv) $r > c$, $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = 0$ (no net enclosed current)

$\Rightarrow B = 0$

d). The given numerical values are, $I = 120\,mA$, $a = 0.04 \times 10^{-2}\,m$, $b = 1.8 \times 10^{-2}\,m$, $c = 2.0 \times 10^{-2}\,m$, $\mu_0 = 4\pi \times 10^{-7}\,NA^{-2}$.

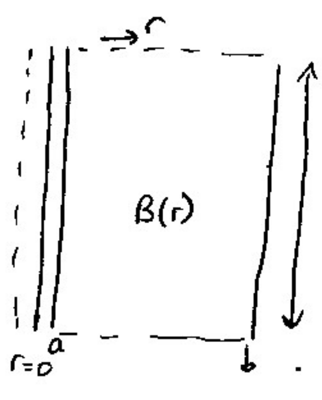
Can plot $B(r)$ over the four regions $\Rightarrow$



$$B_1 = \frac{\mu_0 I}{2\pi a} = 6.0 \times 10^{-5}\,T$$

$$B_2 = \frac{\mu_0 I}{2\pi b} = 1.33 \times 10^{-6}\,T$$

e). For inductance, we find the flux within the cable:



Magnetic Flux, $\phi_B = \iint B \cdot dA$

$$= \iint \frac{\mu_0 I}{2\pi r} \cdot dr \cdot dl$$

$$= \frac{\mu_0 I l}{2\pi} \int_a^b \frac{1}{r}\, dr$$

$$\therefore \quad \phi_B = \frac{\mu_0 I l}{2\pi} \ln(b/a).$$

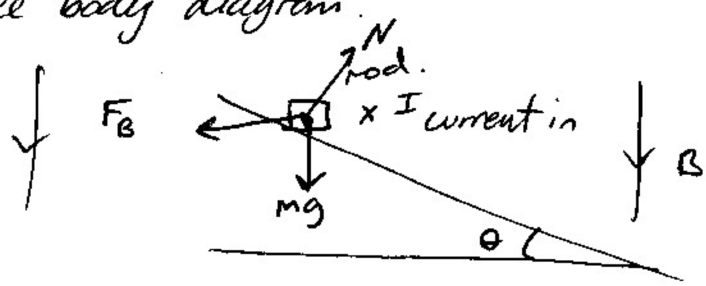
So the inductance is, $L = \frac{\phi_B}{I} = \frac{\mu_0 l}{2\pi} \ln(b/a)$.

and the inductance per m is, $L/l = \frac{\mu_0}{2\pi} \ln(b/a)$

$$= 2 \times 10^{-7} \times \ln\left(\frac{1.8}{0.04}\right).$$

$$= 7.6 \times 10^{-7}\,Hm^{-1}.$$

Q2). a) Free body diagram.



Three forces
- gravity
- magnetic
- normal

b). To produce the force (magnetic) that opposes the rod's motion, the current must flow into the page as shown.
(on the paper, this is away from the viewer)

c) Resolve forces acting down the plane

$$ma = mg\sin\theta - F_B\cos\theta$$

$$\therefore \quad ma = mg\sin\theta - BIL\cos\theta.$$

as the rod accelerates, the rate of change of magnetic flux will increase, inducing greater current.

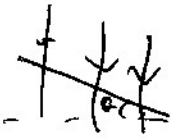
This will continue until equilibrium is reached, so $a = 0$ (no acceleration), and a constant velocity is reached.

d). The steady state, with no net force on the conductor

$$\Rightarrow \quad mg\sin\theta = BIL\cos\theta.$$

$$\therefore \quad I = \frac{mg\sin\theta}{BL\cos\theta}.$$

Now, the emf induced around the loop is.

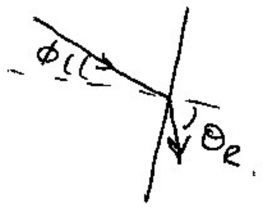


$$\mathcal{E} = IR = \frac{d\Phi_B}{dt}, \text{ but } \quad \phi_B = B\cdot Lx\cos\theta.$$

$$\therefore \quad \mathcal{E} = IR = BL\cos\theta\cdot\frac{dx}{dt} = BLv\cos\theta.$$

$$\text{so.} \quad \frac{mgR\sin\theta}{BL\cos\theta} = BLv\cos\theta \Rightarrow v = \frac{mgR\sin\theta}{B^2L^2\cos^2\theta}.$$

Q3). a) Critical angle, → no light will escape if refracted angle is greater Than 90°



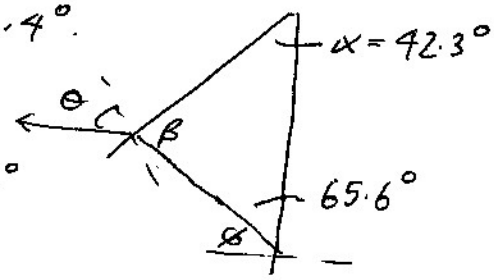
$\Rightarrow$

$\therefore \theta_R = 90°$

Snell's law, $n \sin\phi = \sin\theta_R$

$\therefore \sin\phi = 1/n$.

b). For diamond, $\phi = \sin^{-1}\left(\frac{1}{2.42}\right) = 24.4°$.



⇒ Geometry, $\beta = 180° - 42.3° - 65.6°$
$= 72.1°$.

⇒ Snell's law, $\sin\theta = n\sin(90 - 72.1)$

$\therefore \sin\theta = 2.42 \sin(90 - 72.1)$

$\theta = 48.1°$.

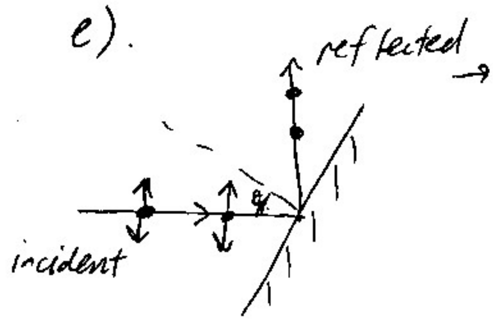
c) Dispersion → The different wavelength's of light experience slightly different refractive indicies.

Different wavelength → different color, are sent in slightly different angles, and if trapped inside crystal for sufficiently long, will be visible as The 'rainbows' seen when looking at jewelry.

d). Brewsters angle, $\tan\theta_p = n$. → $\theta_p = \tan^{-1}(2.42) = 67.5°$

for Diamond.

e).

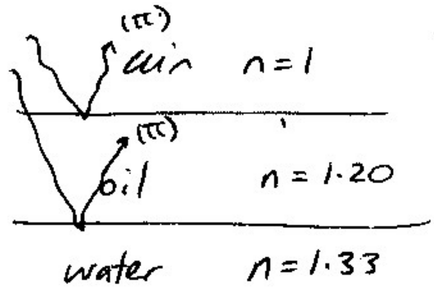


→ Polarised out of the page.

f). Diamond → no birefringence, as crystal is symmetric.
All polarisations will travel at The same speed, regardless of direction.

birefringence requires crystalline asymmetry..

Q4) a). The light will experience a phase change at both reflections, so no net phase change



→ hence, edges will appear bright.

$\Delta OPL = 0$ if thickness is zero, so in phase, constructive interference.

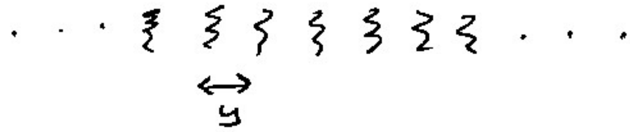
b). Constructive interference, $\Delta OPL = m\lambda$.

Here, $\Delta OPL = 2n_{oil}t$ (no phase changes)

so $t = \frac{m\lambda}{2n_{oil}} = \frac{3 \times 350\,nm}{2 \times 1.2} = 437.5\,nm$.

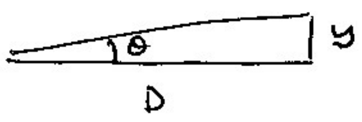
c). The larger the OPL difference, as the two waves travel even greater distances, the light tends to lose coherence. → not coherent waves, Then there is no interference pattern.

Q5). a) Interfence pattern → series of bright and dark friges.



Find the spacing. → Maxima at $d\sin\theta = m\lambda$.

or $\theta \sim \sin\theta \sim \frac{m\lambda}{d}$ (small angle approx.)



$\Rightarrow \theta \sim \frac{y}{D} \sim \frac{m\lambda}{d}$

$\therefore \quad y = \frac{D \times m\lambda}{d} = \frac{2.5 \times 620 \times 10^{-9}}{0.04 \times 10^{-3}}$

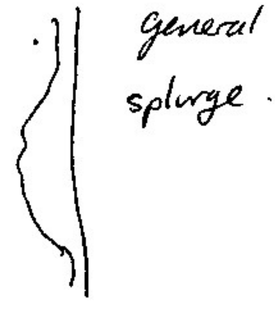
$= 0.0388\,m \simeq 3.9\,cm$. spacing.

b). Interference pattern, with clear maxima and minima. This can only be due to two coherent waves, moving in and out of phase → particles would produce a 'splurge'.

c). Photoelectric Effect - light knocking electrons from a metal. Wave model cannot explain why electron energy is independent of wave intensity but depends on frequency, and also why no time delay for emission.

Black body Radiation - Thermal radiation emitted due to an objects temperature. Planck explained observed spectrum by assuming light is emitted in packets with energy $E = hf$.

d). No interference →



e). Quantum Mechanics.

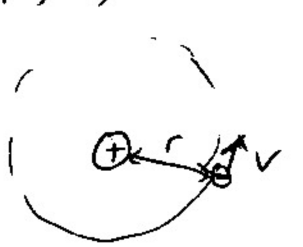
→ prior to detectors → wave function at each slit, $|\psi_{slit1}|^2 = |\psi_{slit2}|^2 = 0.5$.

When detect it, then wave function becomes $|\psi_{slit ?}|^2 = \{ {1 \atop 0}$ (1 if detected, 0 if not), and situation becomes a set of particles leaving a single slit → no interference.

f). $$N = \frac{I}{hf} = \frac{I}{hc/\lambda} = \frac{1.0\times10^{-3}\,J\,s^{-1}m^{-2}}{6.626\times10^{-34}\times3.0\times10^{8}} \times 620\times10^{-9}$$

$$= 3.1\times10^{15} \text{ photons / second / } m^2.$$

Q6) a) Bohr model → planetry model, electron orbits nucleus.



Postulates i) a discrete set of orbits are stable

ii) These are $L = mvr = n\hbar$

iii) Photon emitted if electron changes orbits, $hf = E_{n_1} - E_{n_2}$.

b). Centripetal force is provided by electrostatic force.

$F_e = F_g$

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \Rightarrow \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}.$$

Total energy of orbit (Kinetic + Potential), $E = -\frac{e^2}{4\pi\varepsilon_0 r^2} + \frac{1}{2}mv^2 = -\frac{e^2}{8\pi\varepsilon_0 r}$

Quantisation condition → $mvr = n\hbar$

$\therefore v = \frac{n\hbar}{mr}$

$\therefore$ From centripetal force → $\frac{e^2}{4\pi\varepsilon_0 r} = mv^2$

so that, $\dfrac{e^2}{4\pi\varepsilon_0 r} = m \cdot \dfrac{n^2 h^2}{4\pi^2 m^2 r^2}$

$$r = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2}$$

$\therefore$ Energy, $E_n = -\dfrac{e^2}{8\pi\varepsilon_0 r} = \dfrac{-e^2}{8\pi\varepsilon_0} \cdot \dfrac{\pi m e^2}{n^2 h^2 \varepsilon_0}$

$$\therefore \quad E_n = \frac{-me^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

c). de Broglie $\rightarrow$ stable orbits are those for electron wave is a standing wave $\rightarrow$ integer number of electron wavelengths fit around orbits

$\therefore \quad n\lambda = 2\pi r \quad (n = 1, 2, 3, \ldots)$

or $\quad n\dfrac{h}{p} = 2\pi r$

$$\Rightarrow \quad mvr = n\frac{h}{2\pi}.$$

d). $hf = \dfrac{-me^4}{8\pi\varepsilon_0^2 h^2} \cdot \dfrac{1}{5^2} - \left(-\dfrac{me^4}{8\pi\varepsilon_0^2 h^2} \cdot \dfrac{1}{1^2}\right).$

$\dfrac{hc}{\lambda} = \dfrac{me^4}{8\pi\varepsilon_0^2 h^2}\left(\dfrac{24}{25}\right) \quad \Rightarrow \quad \lambda = \dfrac{8\pi\varepsilon_0^2 h^3 c}{me^4}\left(\dfrac{25}{24}\right)$

$$= 9.50 \times 10^{-8}\,\mathrm{m}.$$

$$\Rightarrow \lambda \approx 95\,\mathrm{nm}.$$

e). Many answers here. For example:

- why a stable orbit was possible with no angular momentum (and so no magnetic moment), s orbitals
- the filling of electron orbitals and the periodic table in general.