1121/1131 Question 1 T2 2012

a) i) $s = 4(0)^2 + 1 = 1m$

ii) $v = \frac{ds}{dt} = 8tms^{-1}$

iii) $a = \frac{d^2s}{dt^2} = 8ms^{-2}$

b) Convert the speed to SI units:

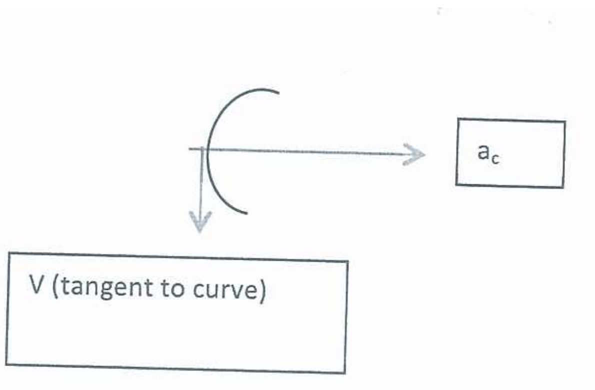
$$50km/hr = \frac{50\times10^3}{3600} = 13.9ms^{-1}$$

Constant acceleration:

$$t = \frac{v}{a} = 13.98 = 1.7s$$

c)
$$x = x_0 + v_0t + \frac{1}{2}at^2$$
$$= 1 + 0 \times 8 + \frac{1}{2} \times 8 \times 1.7^2$$
$$= 13m$$

d) i)



ii) $a_c = \frac{v^2}{r} = \frac{13.9^2}{30} = 6.4ms^{-2}$ towards the center of the curve

e) i) At the same velocity as the car, at a tangent to the curve followed by the car: v = 13.9 ms^{-1} (v in diection shown in diagram above)

ii)
$$y = \frac{1}{2}a_yt^2$$
$$\Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times1.4}{9.8}} = 0.53s$$

iii) $x = v_{x0}t = 13.9 \times 0.53 = 7.4m$

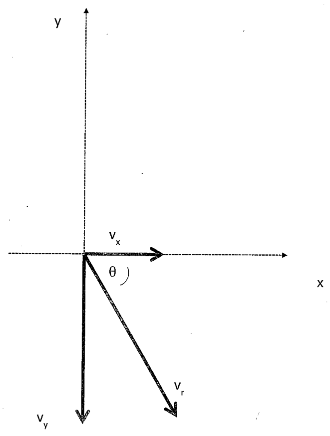
iv) $v_y = a_yt = 9.8 \times 0.53 = 5.2ms^{-1}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{13.9^2 + 5.2^2} = 15ms^{-2}$$

To work out direction need to calculate θ

$$\tan\theta = \frac{v_y}{v_x} = \frac{5.2}{13.9}$$

$\theta = 20.5^o$ below horizontal



1121 Question 2 T2 2012

a) i)

ii) $$F = \frac{Gm_E m_M}{r^2} = m_M a_M$$

$$\Rightarrow a_M = \frac{Gm_E}{r^2} = \frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24}}{(384000 \times 10^3)^2}$$

$$a_M = 2.71 \times 10^{-3} ms^{-2} \text{ towards the Earth}$$

Note: you could also solve it by working out the velocity of the moon using the period and circumference of its orbit and then using $a_c = v^2/r$.

b) i) $$\omega_f = \frac{20000 \times 2\pi}{60} = 2094 rads^{-1}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2094 - 0}{5 \times 60} = 7.0 rads^{-2}$$

ii) $$v_f = \omega_f r = 2094 \times 0.0500 = 104.7 ms^{-1}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{105}{300} = 0.349 ms^{-2}$$

iii) $$\theta = \theta_0 + \omega_i t + \frac{1}{2}\alpha t^2$$

$$= \frac{1}{2} \times 7.0 \times 300^2$$

$$= 315000 rad$$

$$= 50100 rev$$

iv) $$K = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 1.25 \times 10^{-4} \times 2094^2 = 274J$$

v) $$a_c = \frac{v^2}{r} = \omega^2 r^2 r = \omega^2 r = (\frac{10000 \times 2\pi}{60})^2 \times 0.0500 = 54800 ms^{-2}$$

vi) $$a_T = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\frac{d(\omega_0 + \alpha t)}{dt} = r\alpha = 0.0500 \times 7.0 = 0.35 ms^{-2}$$

As $a_T << a_C$ we can ignore a_T. The acceleration is 54 800 ms^{-2} towards the center of the circle.

Q2

i) If non-conservative forces do no work, mechanical energy is conserved.

ii) If external forces are zero (or provide negligible impulse), momentum is conserved.

iii) At the point of maximal compression (or minimum length) of the spring, its length is instantaneously not changing, so the relative velocity of the two masses is instantaneously zero. Hence the two have the same x component of velocity, call it v_1.

Here, no external forces in the x direction, so momentum is conserved.

Considering the initial state and that at maximal spring compression:

p_x conserved: $mv_0 = 2mv_1$.

so $v_1 = v_0/2$ (only 1 mark for guessing this)

Here, there are no non-conservative forces so mechanical energy is conserved.

E conserved: $U_{initial} + K_{initial} = U_{final} + K_{final}$

$$½mv_0^2 = ½*2mv_1^2 + ½kx^2 = mv_1^2 + ½kx^2$$

$$½mv_0^2 = m(v_0/2)^2 + ½kx^2 = mv_0^2/4 + ½kx^2$$

$$mv_0^2/4 = ½kx^2$$

$$x = \sqrt{\frac{m}{2k}}\,v_0$$

iv) Here, there are no non-conservative forces, and no external forces in the x direction, so both mechanical energy and momentum are conserved.

Considering the states before and after the collision:

p_x conserved: $mv_0 = mV + mv$ (1) or equivalent

E conserved: $U_{initial} + K_{initial} = U_{final} + K_{final}$

$$0 + ½mv_0^2 = 0 + ½mV^2 + ½mv^2 \quad (2)$$

(1) gives $V = (v_0 - v)$ substitute in (2) gives

$$v_0^2 = (v_0 - v)^2 + v^2$$

$$v_0^2 = v_0^2 - 2vv_0 + 2v^2$$

$$0 = -2vv_0 + 2v^2$$

$$0 = v(v - v_0)$$

So either $v = 0$ or $v = v_0$.

and (respectively) $V = v_0$ or $V = 0$

v) There are two solutions. If there really is a collision, the velocities will change, so the first solution above ($v = 0$ and $V = v_0$) applies. The second solution ($v = v_0$ and $V = 0$) corresponds to the case where the objects miss each other, so momentum and mechanical energy are both conserved.

vi) If the system comprises N particles with masses m_i at positions $\underline{r}_i$, the centre of mass is a point whose position is $\underline{r}_{cm} = \frac{\sum_{i=1}^{N} m_i r_i}{\sum_{i=1}^{N} m_i}$ OR $\underline{r}_{cm} = \frac{\sum_{i=1}^{N} m_i r_i}{\text{total mass}}$.

vii) Taking time derivatives of the expression in (vi) and noting that the masses are constant:

$$\underline{v}_{cm} = \frac{\sum_{i=1}^{N} m_i v_i}{\sum_{i=1}^{N} m_i} .$$

Apply this to the condition before the collision and we have

$$\underline{v}_{cm} = \frac{\sum_{i=1}^{2} m_i v_i}{\sum_{i=1}^{2} m_i} = \frac{mv_0 + 0}{2m} = v_0/2$$

Apply this to the condition of maximum compression gives

$$\underline{v}_{cm} = \frac{\sum_{i=1}^{2} m_i v_i}{\sum_{i=1}^{2} m_i} = \frac{2mv1}{2m} = v_1$$ and, substituting from part (iii)

$$\underline{v}_{cm} = v_0/2$$

viii) Newton's second law for an extended object or system is

$\mathbf{F}_{ext} = (\text{total mass})\mathbf{a}_{cm}$ Here, the external force is zero so the velocity of the centre of mass is constant.

OR

Newton's first law for an extended system is that, if $\mathbf{F}_{ext} = 0$, the velocity of the centre of mass is constant.

OR

Some other equivalent statement.

Question 4 1121 T2 2012

a) i) 3

ii)

$$P = \frac{F}{A}$$

$$P_f = P_i + \frac{mg}{A} = 1.17 \times 10^5 + \frac{5 \times 9.8}{12.0 \times 10^{-4}} = 1.58 \times 10^5 Pa$$

iii)

$$PV = \text{const}$$

$$P_i V_i = P_f V_f$$

$$1.17 \times 10^5 \times 0.20 \times 12.00 \times 10^{-4} = 1.58 \times 10^5 \times h_f \times 12 \times 10^{-4}$$

$$h_f = 14.8cm$$

b) i)

$$P = kA|\frac{dT}{dx}|$$

P is constant along the rod

$$\Rightarrow P = 385 \times 10 \times 10^{-4} \times \frac{(70-45)}{1.00} = 9.63W$$

ii)

$$9.625 = 50.2 \times 10.0 \times 10^{-4} \times \frac{(45-15)}{L_2}$$

$$L_2 = 0.156m$$

c) i) $\Delta E_{int} = 0$ as isothermal

ii) $\Delta E_{int} = Q + W$

$W = 0$ as volume does not change

$\Delta E_{int} = -233kJ$

iii)

$$W_{C \to A} = \Delta E_{intC \to A} - Q_{C \to A}$$

$Q_{C \to A} = 0$ a adiabatic

$$\Delta E_{intC \to A} = -(\Delta E_{intA \to B} + \Delta E_{intB \to C})$$

as no change in internal energy over cycle

$$= -(0 - 233)$$

$$= 233kJ$$

$$W_{C \to A} = 233kJ$$

Alternatively could use $W = -\int PdV$ and integrate the expression.

This takes longer.

PHYS1121 Exam 2012-T2

Oscillation question [25] marks

(a) (i) At the moment of release:

$$\sum F = -kx = ma$$

$$125 \times 0.687 = 5.0a$$

$$a = \frac{85.9}{5.0} = 17.17 = 17.2 \mathrm{m/s^2}$$

(ii) the period of oscillation for the spring system:

$$T = 2\pi\sqrt{\frac{m}{k}} = 6.28 \times \sqrt{\frac{5.00}{125}} = 1.26 \text{ s}$$

(iii) The work done by the spring between x = 0.687 and 0:

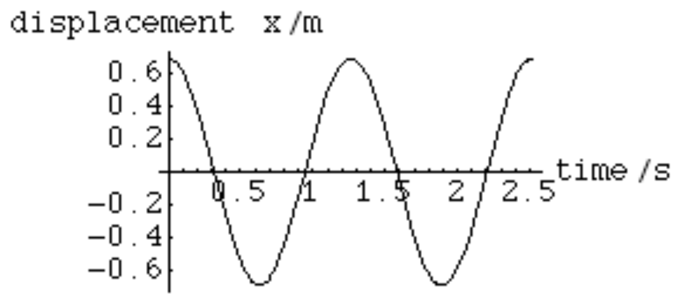
$$W = F.x = \int_0^{0.687} kx dx = \frac{1}{2}kx^2 = 29.5 \text{ J}$$

(iv)

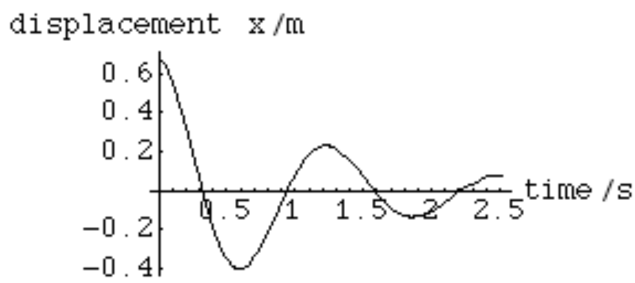
$$f = \frac{1}{T} = 0.794 \text{ Hz}$$

$$2T = 2.52 \text{ s}$$

$$\text{displacement } x = 0.687\cos(2\pi ft)$$



Small damping (b = 0.35 – not for marking)



(b) (i) If the path difference results in two periods arrival difference for the wave trains from each source, it must be two wavelengths long (leading to constructive interference)

$$S2P - S1P = 13 - 12 = 1\text{m}$$
$$\text{thus } \lambda = 1/2 = 0.5 \text{ m}$$
$$v = \lambda f$$
$$f = \frac{v}{\lambda} = \frac{350}{0.5} = 700 \text{ Hz}$$

(ii) For 1.5 periods difference, the new path difference must be 1.5 wavelengths long (which does lead to destructive interference). Both distances S1P and S2P will increase by s1 and s2.

$$(S2P + s2) - (S1P + s1) = 0.75 \quad \text{eqn(1)}$$
from Pythagoras:
$$(S2P + s2)^2 = 25 + (S1P + s1)^2 \quad \text{eqn(2)}$$
$$(S2P + s2) = 12.75 + s1$$
Substitute into eqn(2)
$$(12.75 + s1)^2 = (12 + s1)^2 + 25$$
which can be solved for s1

The answer is then $12 + s1$

(c) Have to do (ii) before (i) to find the train speed.

(ii) If the frequency appears lower to the observer, the source and the observer must be moving away from each other: their velocities will be negative

$$f' = \left(\frac{v + v_o}{v - v_s}\right) f$$
$$558 = \left(\frac{340 - 10}{340 + x}\right) \times 600$$
$$x = \frac{600}{558} \times 330 - 340 = 14.8 \approx 15\text{m/s}$$

(i) Train moving away from stationary observer:

$$f' = \left(\frac{v}{v + v_s}\right) f = \left(\frac{340}{355}\right) 600 = 575 \text{ Hz}$$