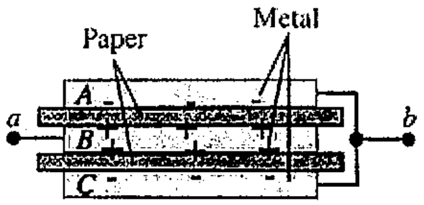
Answer 1

(a) The diagram should look something like this, i.e., an equal number of +ve and -ve charges on the metal surfaces on either side of the paper.



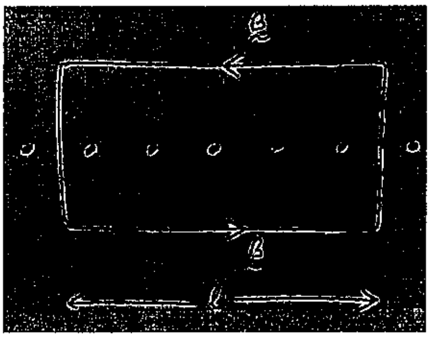
and the student should make it very clear in writing that the charges are on the surface of the metal plates

(b) The arrangement is equivalent to two identical capacitors in parallel [5 marks]. The total capacitance is therefore twice the capacitance of a single capacitor:

$$
\begin{aligned}
C &= 2\epsilon_0\kappa\frac{A}{d} \\
&= \frac{2 \times 8.85 \times 10^{-12} \times 4.2 \times (15.0 \times 10^{-2})^2}{0.45 \times 10^{-3}} \\
&= 3.72\,\mathrm{nF}
\end{aligned}
$$

Answer 2

(a) By symmetry, and the right hand rule, the magnetic field must be as shown in the diagram below.



Use an Amperian loop as shown The contribution to the integral of $\boldsymbol{B}.\boldsymbol{ds}$ from the vertical segments is zero. The contribution from the horizontal segments is $2Bl$. This equals the enclosed current, which is Il/a. Using Ampere's Law:

$$2Bl = \frac{\mu_0 Il}{a}$$

or

$$B = \frac{1}{2a}\mu_0 I$$

Above the plane, the magnetic field is in the $-y$ direction, so

$$\boldsymbol{B} = -\frac{1}{2a}\mu_0 I\hat{\jmath}$$

where $\boldsymbol{B}$ and $\hat{\jmath}$ are vectors.

(b) The only change from part (a) is that the magnetic field is now in the $+y$ direction:

$$\boldsymbol{B} = \frac{1}{2a}\mu_0 I\hat{\jmath}$$

where $\boldsymbol{B}$ and $\hat{\jmath}$ are vectors.

(c) This is essentially the same problem as for part (a). The total current flowing through the Amperian loop is now σlv[3 marks]. So,

$$2Bl = \mu_0\sigma lv,$$

therefore

$$\boldsymbol{B} = \frac{1}{2}\mu_0\sigma v\hat{\jmath}$$

where $\boldsymbol{B}$ and $\hat{\jmath}$ are vectors.

Answer 3

(a) The induced emf ϵ is the rate of change of magnetic flux through the circuit, which is B times the rate of change of area of the loop:

$$\epsilon = -\frac{d\phi_b}{dt} = vlB = 7.50 \times 0.500 \times 0.800 = 3.00V$$

(b) By Lenz's Law, the induced emf must oppose the change in magnetic flux, i.e., it should cause a magnetic field out of the page. By the right-hand-rule this means the current should be counter-clockwise, i.e., from b to a

(c) The induced emf causes a current to flow in the rod given by

$$I = \frac{V}{R} = \frac{3.00}{1.50} = 2.00A$$

This current feels a force from the magnetic field of

$$F = ilB = 2.00 \times 0.500 \times 0.800 = 0.800N$$

(d)

$$Fv = 0.800 \times 7.50 = 6.00J$$
$$I^2R = 2.00^2 \times 1.50 = 6.00J$$

So the two rates of work are the same, as expected

Answer 4

(a) The volume of copper available is clearly m/ρ. This must equal the volume of wire, which is $\pi {r_w}^2 l_w$, therefore

$$l_w = \frac{m}{\pi {r_w}^2 \rho}$$

(b) The current flowing through the wire is by Ohm's Law V/R, and $R = l_w/\sigma\pi {r_w}^2$, therefore

$$I = \frac{V\sigma\pi {r_w}^2}{l_w} = \frac{V\sigma\pi^2\rho {r_w}^4}{m}$$

where we have used the result from part (a).

(c) The length of the solenoid is

$$l_s = \text{number of turns of wire} \times 2r_w,$$

since each turn is $2r_w$ wide. Now, the number of turns of wire is just the total length of the wire divided by the circumference of a turn, i.e., $l_w/2\pi r_s$. Inserting this into the equation for l_s we get

$$l_s = \frac{l_w}{2\pi r_s} \times 2r_w = \frac{l_w r_w}{\pi r_s}$$

(d)

$$l_s = \frac{l_w r_w}{\pi r_s} = \frac{m}{\pi^2 \rho r_w r_s}$$

(e) The number of turns per unit length, n, is the reciprocal of the length of each turn, i..e,

$$n = \frac{1}{2r_w}$$

(f) The magnetic field in the solenoid is

$$B = \mu_0 n I = \frac{\mu_0 I}{2r_w} = \frac{\mu_o V\sigma\pi^2\rho {r_w}^3}{2m}$$

(g) The total energy stored in the magnetic field, U, is

$$\begin{aligned}
U &= \frac{B^2}{2\mu_0} \times \text{volume of the solenoid} \\
&= \frac{B^2}{2\mu_0} \times l_s \pi {r_s}^2 \\
&= \frac{{\mu_0}^2 V^2\sigma^2\pi^4\rho^2 {r_w}^6}{4m^2 2\mu_0} \times \frac{m}{\pi^2\rho r_w r_s} \times \pi {r_s}^2 \\
&= \frac{\mu_0 V^2\sigma^2\pi^3\rho {r_w}^5 r_s}{8m}
\end{aligned}$$

Q.5

(a) $\overline{S} = 1.3 \times 10^3\ \mathrm{Wm}^{-2} \;=\; \dfrac{1}{\mu_0}\overline{\mathbf{E}\times\mathbf{B}} \;=\; \dfrac{E_{rms}^2}{\mu_0 c}$

So $E_{rms}^2 \;=\; 1.3 \times 10^3 \times 4\pi \times 10^{-7} \times 3 \times 10^8 \;=\; 4.9 \times 10^5$

so $E_{rms} \;=\; 700\ \mathrm{Vm}^{-1}$

$$B_{rms} = \frac{E_{rms}}{c} = \frac{700}{3 \times 10^8} = 2.3 \times 10^{-6}\ \mathrm{T}$$

(b) Pressure = rate of change of momentum per unit area.
Momentum/sec carried by 1.3 kW of sunlight = 1300/c SI units.

So, force on each 10^{-2} kg $= \dfrac{1300}{c}$

So, acceln $= \dfrac{10^2 \times 1300}{3 \times 10^8} = 4.3 \times 10^{-4}\ \mathrm{ms}^{-2}$

(NB This would take about 4 years to reach escape velocity from the Sun…)

Q.6

(a) $f = \dfrac{c}{\lambda} = \dfrac{3 \times 10^8}{633 \times 10^{-9}} = 4.74 \times 10^{14}$ Hz(or 474 THz)

(b) $f = \dfrac{c}{\lambda} = \dfrac{c_{glass}}{\lambda_{glass}}$ so $\lambda_{glass} = \lambda \dfrac{c_{glass}}{c} = \dfrac{\lambda}{n} = 422$ nm

(c) $d \sin\theta = m\lambda$ so $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{633 \times 10^{-9}}{10^{-6}} = 0.633$ *ie* $\theta_1 = 39.3^0$

Q.7

(a) Energy of a 550 nm photon $= hf = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}}$

$= 3.62 \times 10^{-19}$ J

Energy radiated per second = 0.8 W, so no. of photons per second $= \dfrac{0.8}{3.62 \times 10^{-19}}$

$= 2.2 \times 10^{18}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.8 \times 1.6 \times 10^{-19}} = 691$ nm

Q.8

(a) Outside: $\frac{1}{2} m_e v_1^2 = 100 \times 1.6 \times 10^{-19}$ J,so $v_1^2 = \dfrac{1.6 \times 10^{-17}}{0.5 \times 9.11 \times 10^{-31}} = 0.3515 \times 10^{14}$

$v_1 = 0.593 \times 10^7 = 5.93 \times 10^6\ \mathrm{ms}^{-1}$

Inside, the KE is increased from 100 to 115 eV, so $v_2 = 5.93 \times 10^6 \sqrt{\dfrac{115}{100}} = 6.36 \times 10^6\ \mathrm{ms}^{-1}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$, where $\dfrac{n_1}{n_2} = \dfrac{v_2}{v_1} = \dfrac{6.36}{5.93} = 1.0725$

The velocity is greater inside, so the refracted angle is greater than the incident angle:

$\sin\theta_2 = 1.0725 \times \dfrac{1}{\sqrt{2}} = 0.7584$, so $\theta_2 = 49.3^0$.