**SOLUTION for PHYS1231 Final Exam S2 2009**

**Total for 5 Questions: 80 marks**

**For formula sheet:**

$E_{max} = cB_{max}$ $\quad$ $I = S_{av} = \dfrac{E_{max}^2}{2\mu_0 c} = \dfrac{cB_{max}^2}{2\mu_0}$ $\quad$ $I = S_{av} = cu_{av}$ $\quad$ $P = \dfrac{2I}{c}$

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{1}{n^2}$$

**Question 1. EM Waves (Marks 22)**

(a) An electromagnetic (EM) sinusoidal plane wave with frequency $f = 90.0$ MHz propagates in the $+x$ direction. The electric field of the EM wave has a peak value $E = 2\text{mV/m}$ directed along the $\pm y$ direction.

(i) Find the wavelength, period and the maximum value of the magnetic field for this EM wave.

(ii) Write down expressions for the space and time variations of the electric and magnetic fields; give the expressions in SI units. Include in your expressions the appropriate unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$.

(iii) Find the average power per unit area carried by this wave.

(iv) Find the average energy density in the radiation.

**Solution:**

For this EM wave

$$f = 90\text{ MHz},\quad E_{max} = 2.00\text{x}10^{-3}\text{ Vm}^{-1}$$

(i) $$\lambda = \frac{c}{f} = \frac{3\text{x}10^8}{90\text{x}10^6} = 3.33\text{ m}$$

$$T = \frac{1}{f} = 1.1\text{x}10^{-8}\text{ s} = 11.8\text{ ns}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{2.00\text{x}10^{-3}}{3\text{x}10^8} = 6.67\text{x}10^{-12}\text{ T}$$

(ii) The EM wave is represented in the standard form:

$$E = E_{max}\cos(kx - \omega t)$$

using the substitution $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f = \frac{2\pi}{T}$ we can express this as

$$E = E_{max}\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$B = B_{max}\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

so that

$$\mathbf{E} = (2.00\ \text{mV/m})\cos 2\pi\left(\frac{x}{3.33\ \text{m}} - \frac{t}{11.1\ \text{ns}}\right)\hat{\mathbf{j}}$$

$$\mathbf{B} = (6.67\ \text{pT})\cos 2\pi\left(\frac{x}{3.33\ \text{m}} - \frac{t}{11.1\ \text{ns}}\right)\hat{\mathbf{k}}$$

(iii) $$I = S_{av} = \frac{E_{max}^2}{2\mu_0 c} = \frac{(2.00x10^{-3})^2}{2(4\pi x10^{-7})(3x10^8)} = 5.30x10^{-9}\ \text{Wm}^{-2}$$

(iv)

$$I = cu_{av}$$

$$u_{av} = \frac{I}{c} = \frac{5.30x10^{-9}}{3x10^8} = 1.77x10^{-17}\ \text{Jm}^{-3}$$

(b) Light with free space wavelength $\lambda = 780nm$ travels a distance $2x10^{-6}m$ in a transparent medium of refractive index 1.6. Calculate,

(i) the optical path length

(ii) the wavelength of the light in the transparent medium

(iii) the phase difference after travelling the distance $2x10^{-6}m$ with respect to light travelling the same distance in free space.

**Solution:**

(i) the optical path length (o.p.l) is

o.p.l = (physical distance travelled in medium) x (refractive index)

$= (2x10^{-6}\ \text{m})(1.6) = 3.2x10^{-6}$

(ii) The wavelength in the medium is $\lambda_m$ given by

$$\lambda_m = \frac{\lambda_0}{n} = \frac{780\ \text{nm}}{1.6} = 487.5\text{nm}$$

where $\lambda_0$ is the free space (vacuum) wavelength.

(iii) The physical distance travelled is $l = 2.0x10^{-6}\text{m}$

The refractive indices are: $n_{medium} = 1.6, n_{vacuum} = 1.0$

$\therefore$ path difference $= (n_{medium} l - n_{vacuum} l)$

$$= (1.6)(2.0\text{x}10^{-6}) - (1.0)(2.0\text{x}10^{-6}) = 1.2\text{x}10^{-6} \text{ m}$$

Phase difference = 2π(path difference/wavelength)

$$= 2\pi\left(\frac{1.2\text{x}10^{-6}}{780\text{x}10^{-9}}\right) = 3.08\pi \text{ radians}$$

**Alternative method (as per tut)**

$$\text{phase difference} = 2\pi\left[\frac{l}{\lambda} - \frac{l}{\lambda_0}\right] = 2\pi\left[\frac{2.0x10^{-6}}{487.5\text{x}10^{-9}} - \frac{2x10^{-6}}{780\text{x}10^{-9}}\right] = 2\pi[4.1 - 2.56]$$

$$= 2\pi(1.54) = 3.08\pi \text{ radians}$$

$$= 2\pi \frac{8 * 10^{-7}}{600 * 10^{-9}} = \frac{8\pi}{3} radians$$

$$= (2\pi + \frac{2}{3}\pi) radians$$

or phase difference is $2\pi\left[\frac{L}{\lambda} - \frac{L}{\lambda_0}\right] = 8\pi / 3$ on substitution

**Question 2. (Marks 15)**

(a) Consider a Young's double slit apparatus in which the centre-to-centre slit spacing is 0.3mm and the slits-to-screen distance is 0.8m. Two wavelengths of light $\lambda_1, \lambda_2$, illuminate the slits simultaneously, where $\lambda_1 = 500nm$ and $\lambda_2 = 600nm$, producing two interference patterns on the screen. Find the separation (distance) on the screen between the two third-order interference patterns produced by $\lambda_1, \lambda_2$.

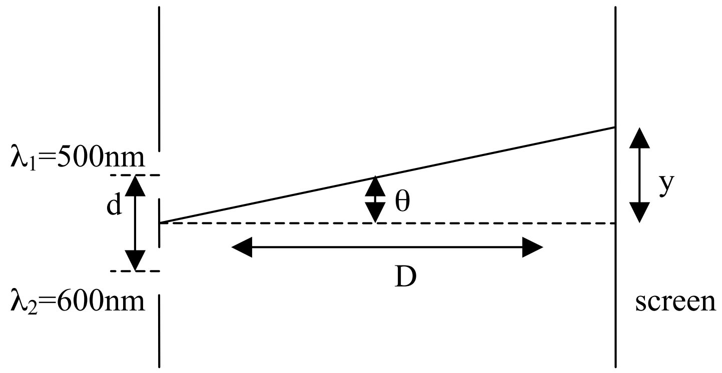
**Solution:**

*This question involves interference only (no diffraction)*

Maxima (bright fringes) at $d \sin\theta = m\lambda$

Minima (dark fringes) at $d \sin\theta = (m + \frac{1}{2})\lambda$

m = <u>order</u> of interference = 0, 1, 2, 3, …



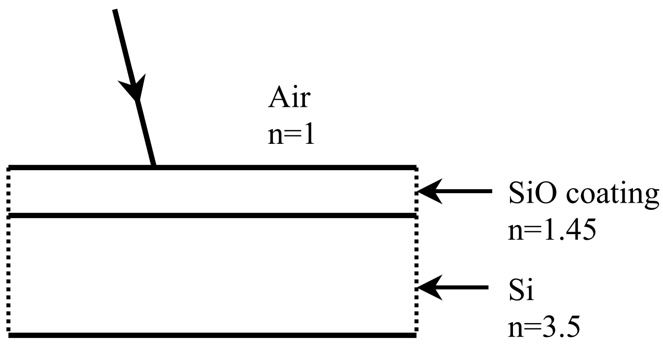

$d = 0.3\,mm = 3\text{x}10^{-4}\,m, D = 0.8\,m$

$3^{rd}$ Order fringes ⇒ m = 3.

$$\sin\theta_1 = \frac{3*500*10^{-9}}{3*10^{-4}} = 5.0*10^{-3}$$
$$\sin\theta_2 = \frac{3*600*10^{-9}}{3*10^{-4}} = 6.0*10^{-3}$$
$$\tan\theta = \frac{y}{D}, D = 0.8m$$
for a small angle $\tan\theta \approx \sin\theta$
$$\therefore y_2 - y_1 = D(\sin\theta_2 - \sin\theta_1) = 8.0*10^{-4}\,m$$

(b) Solar cell non-reflective coating (e.g. example 37.4 p1063)
To maximize collection efficiency by minimizing reflective losses, the surface of silicon (Si) solar cells can be coated with a thin film of silicon monoxide (SiO).

(i) On the diagram below, representing a SiO coated Si solar cell, sketch in transmitted and reflected rays, indicating on the sketch all phase changes occurring at the air-SiO and SiO-Si interfaces.

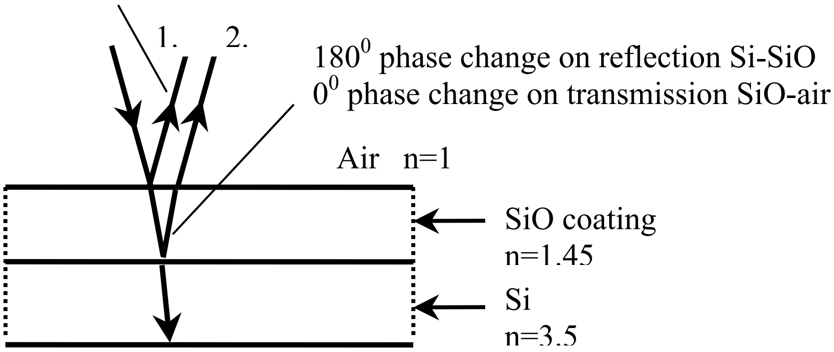



(ii) For the SiO coated Si solar cell, calculate the minimum film thickness required to minimize reflection losses of solar radiation of wavelength $\lambda = 550nm$.

(Refractive indices: Silicon cell: $n_{Si} = 3.5$; Coating, $n_{SiO} = 1.45$)

**Solution:**

(i) $180^0$ phase change on reflection SiO-air
$0^0$ phase change on transmission air-SiO




(ii) Minimum reflection occurs when exiting rays 1. and 2. interfere destructively.

Rays 1. and 2. *both* undergo a $180^0$ phase change upon reflection, i.e. no *net* change on reflection.

A phase change of ray 2. w.r.t. ray 1. occurs in transmission a distance 2t within the SiO, where t is the SiO film thickness.

$$\therefore \quad 2nt = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4n} = \frac{550x10^{-9}}{4(1.45)} = 9.48x10^{-8} \text{ m} \approx 95 \text{ nm}$$

**Question 3 (Marks 10)**
A student attends a physics lecture with a camera to record the notes the lecturer has projected on to the theatre's screen. The camera has a lens diameter 2mm. The lecturer has written the notes in blue ink ( $\lambda_{blue} = 450nm$ ) and each character (letter or symbol) can be considered to be a 3mm diameter circle on the screen.

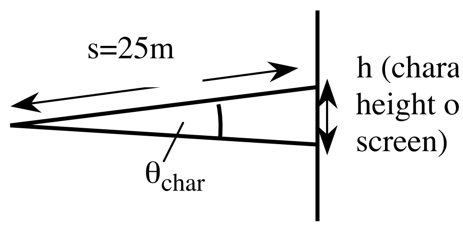
(i) Will the camera resolve individual characters on the screen if the student sits at the back of the theatre, at a distance of 25 m from the screen? If not,

(ii) what is the minimum distance the camera must be from the screen such that individual characters are just resolvable?

Note: a plain yes/no answer will obtain no marks; the principle and argument used to show resolvability must be given, with a simple sketch if appropriate, along with all working in your calculation.

**Solution:**

(a) The camera has lens diameter $D = 2\text{mm}$. The wavelength of light from the screen is $\lambda = 450\text{nm}$. Characters on the screen have height $h = 3\text{mm}$.



(i) If the student is 25 m from the screen we have:

$$\theta_{char} = \frac{h}{s} = \frac{3x10^{-3}\,m}{25m} = 1.2x10^{-4}\,rad$$

The Rayleigh Criterion gives

$$\theta r = 1.22\frac{\lambda}{D} = 1.22\frac{450x10^{-9}}{2x10^{-3}} = 2.7x10^{-4}\,rad$$

$\theta r = 2.7x10^{-4}\,rad$ is the angle subtended by the character on the screen at the viewer's position which allows resolution of individual characters by the camera lens, when resolution is limited by diffraction. At 25 m the character height is too small to allow resolution.

(ii) For characters to be just resolvable by the camera lens we must have

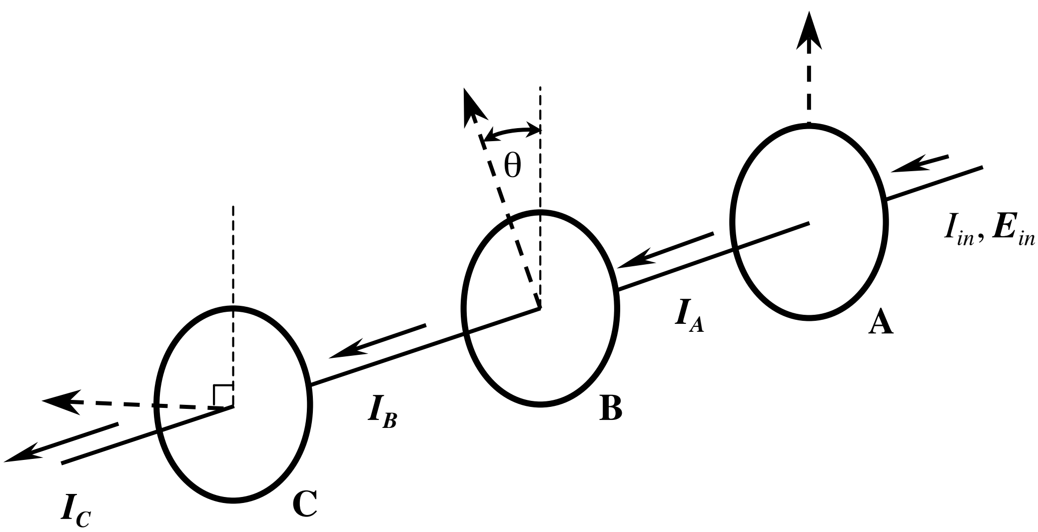
$$\theta_{char} = \theta r = 2.7x10^{-4}\,rad$$

so the student must sit at, or closer than,

$$s = \frac{h}{\theta r} = \frac{3x10^{-3}}{2.7x10^{-4}} = 11.1m$$

**Question 4 (Marks 11)**

Three ideal polarizing sheets are arranged as shown below. Unpolarised light of intensity $I_{in}$ is incident upon polarising sheet A. The polarisation axes of the sheets are indicated by the broken arrows. Sheets A and C are arranged as shown with their axes of polarization at 90°, sheet C is rotated such that its polarization axis is at angle θ to the vertical (and therefore also at θ° to the polarization axis of sheet A).



(i) What is the intensity of light transmitted by sheet A?
(ii) If $\theta = 45^\circ$, find the intensity of light transmitted by each of the three sheets and therefore that transmitted by the system of sheets A, B and C.
(iii) If $\theta = 30^\circ$, find the intensity of light transmitted by the system.

**Solution:**
(i) What is the intensity of light transmitted by sheet A?

When polariser A absorbs one of the components (e.g. the z-component, see diagram above) half the incident intensity is removed. Therefore,

$$I_A = \frac{1}{2} I_{in}$$

(ii) If $\theta = 45^\circ$, find the intensity of light transmitted by each of the three sheets and therefore that transmitted by the system of sheets A, B and C.

$$I_A = I_{in}/2 \text{ and using Malus' law,}$$

$$I_B = I_A \cos^2\theta = I_A \cos^2(45^\circ) = I_A/2 = I_{in}/4\,,$$

$$\therefore \qquad I_c = (I_{in}/4).\cos^2(45^\circ) = I_{in}/8$$

(iii) If $\theta = 30^\circ$, find the intensity of light transmitted by the system of sheets A, B and C.

$$I_A = I_{in}/2$$

and

$$I_B = I_A \cos^2\theta = I_A \cos^2(30^\circ) = 3I_A/4 = 3\,I_{in}/8$$

$$I_c = (3I_{in}/8).\cos^2(60^\circ) = 3I_{in}/32$$

**Question 5 (22 Marks)**

(a) Find the de Broglie wavelength of a 25kV electron.

**Solution:**

$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.6x10^{-19})25x10^3}{9.11x10^{-31}}} = 9.37x10^7\ ms^{-1}$$

- -this is a non-relativistic case.

$$\lambda = \frac{h}{p} = \frac{6.63x10^{-34}}{(9.11x10^{-31})(9.37x10^7)} = 7.77x10^{-12}\ m \quad (= 7.77pm)$$

(b) The lifetime of the unstable hydrogen $n = 2$ state is approximately 10ns. Using Heisenberg's Uncertainty Principle determine the number of significant figures that may be used to express its energy.

**Solution:**

**Solution:**

Heisenberg: $\Delta E.\Delta t = h/2\pi$.

The hydrogen energy levels are

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{1}{n^2}$$

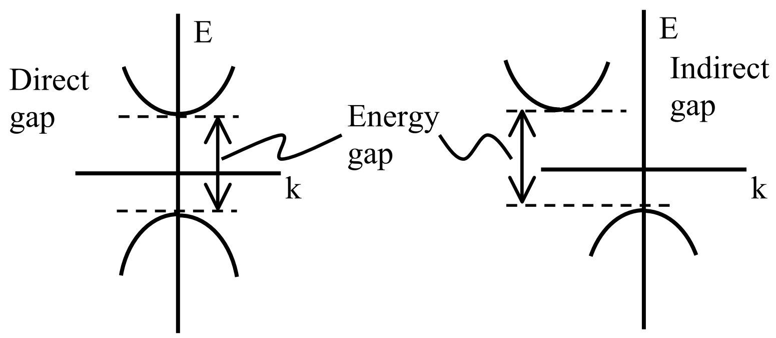
For n = 2 this gives E = -3.4 eV

$\Delta E/E = \dfrac{h}{2\pi\Delta t.E} \approx 10^{-9}$ on substitution E = -3.4 eV, t = 10ns 9 significant figures only

(b) Provide a simple *labelled* sketch showing the fundamental difference between a direct and an indirect gap semiconductor. Name one semiconductor material of each type.

**Solution:**

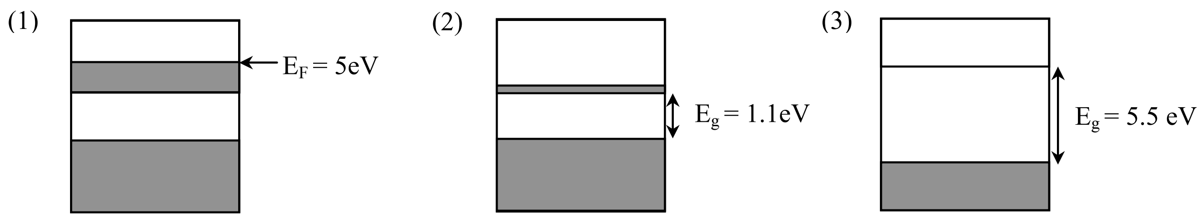
In a direct gap semiconductor the minimum in conduction band is directly above maximum in valence band (at the same value of wavevector k). In an indirect gap semiconductor this is not the case:



Names of direct gap and indirect gap semiconductors. The obvious examples are (although students may cite several others):

Silicon – indirect gap
Gallium Arsenide (GaAs) – direct gap

(d) Three materials have the energy band structures shown schematically in the diagram below representing, (1) a metal, (2) an n-type doped semiconductor and (3) an insulator. The shaded areas indicate occupied (by electrons) energy ranges.




(i) For the metal shown in (1), find the Fermi velocity and the thermal velocity of the electrons at 300K.
(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2).
(iii) By comparing the energy gap values for materials (2) and (3) state, with your reasoning, whether material (3) is expected to be transparent or opaque to visible light at room temperature. (The visible region of the EM spectrum spans the wavelength range λ=400nm to λ=700nm approx.)

**Solution**

(a) (i) Fermi velocity

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2(5.0x10^{-19}J)}{9.1x10^{-31}kg}} \approx 1.1x10^6\ ms^{-1}$$

Thermal velocity

$$v_{th} = \sqrt{\frac{2k_B T}{m_e}}$$

$$= \sqrt{\frac{2\left(1.38x10^{-23}\ JK^{-1}\right)\left(300K\right)}{9.1x10^{-31}}}$$

$$\approx 1x10^{5}\,ms^{-1}$$

(ii)

$$\lambda_{g} = \frac{\mathrm{hc}}{\mathrm{E_g}} = \frac{\left(6.63\mathrm{x}10^{-34}\right)\left(3\mathrm{x}10^{8}\right)}{\left(1.1\right)\left(1.6\mathrm{x}10^{-19}\right)} = 1.1\mathrm{x}10^{-8}\mathrm{m}$$

(iii) Material (2) is opaque because the band gap magnitude permits absorption of all wavelengths in the visible spectrum (400nm-700nm). Material (3) is transparent because the  band gap magnitude means visible wavelengths are not absorbed.