

SOLUTION for PHYS1221 Final Exam S2 2009

Total for 5 Questions: 60 marks

For formula sheet:

$$E_{\max} = cB_{\max} \quad I = S_{av} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0} \quad I = S_{av} = cu_{av} \quad P = \frac{2I}{c}$$
$$E_n = -\left(\frac{me^4}{8\epsilon_0^2 h^2}\right) \frac{1}{n^2}$$

Question 1. EM Waves (Marks 14)

- (a) An electromagnetic (EM) sinusoidal plane wave with frequency $f = 90.0$ MHz propagates in the $+x$ direction. The electric field of the EM wave has a peak value $E = 2\text{mV/m}$ directed along the $\pm y$ direction.
- (i) Find the wavelength, period and the maximum value of the magnetic field for this EM wave.
- (ii) Write down expressions for the space and time variations of the electric and magnetic fields; give the expressions in SI units. Include in your expressions the appropriate unit vectors $\hat{i}, \hat{j}, \hat{k}$.
- (iii) Find the average power per unit area carried by this wave.

Solution:

For this EM wave

$$f = 90 \text{ MHz}, \quad E_{\max} = 2.00 \times 10^{-3} \text{ Vm}^{-1}$$

$$(i) \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{90 \times 10^6} = 3.33 \text{ m}$$

$$T = \frac{1}{f} = 1.1 \times 10^{-8} \text{ s} = 11.8 \text{ ns}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{2.00 \times 10^{-3}}{3 \times 10^8} = 6.67 \times 10^{-12} \text{ T}$$

- (ii) The EM wave is represented in the standard form:

$$E = E_{\max} \cos(kx - \omega t)$$

using the substitution $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f = \frac{2\pi}{T}$ we can express this as

$$E = E_{\max} \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right)$$

$$B = B_{\max} \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right)$$

so that

$$\mathbf{E} = (2.00 \text{ mV/m}) \cos 2\pi \left(\frac{x}{3.33 \text{ m}} - \frac{t}{11.1 \text{ ns}} \right) \hat{\mathbf{j}}$$

$$\mathbf{B} = (6.67 \text{ pT}) \cos 2\pi \left(\frac{x}{3.33 \text{ m}} - \frac{t}{11.1 \text{ ns}} \right) \hat{\mathbf{k}}$$

$$(iii) \quad I = S_{\text{av}} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{(2.00 \times 10^{-3})^2}{2(4\pi \times 10^{-7})(3 \times 10^8)} = 5.30 \times 10^{-9} \text{ Wm}^{-2}$$

(b) Light with free space wavelength $\lambda = 780 \text{ nm}$ travels a distance $2 \times 10^{-6} \text{ m}$ in a transparent medium of refractive index 1.6. Calculate,

- (i) the optical path length
- (ii) the wavelength of the light in the transparent medium

Solution:

- (i) the optical path length (o.p.l) is

$$\begin{aligned} \text{o.p.l} &= (\text{physical distance travelled in medium}) \times (\text{refractive index}) \\ &= (2 \times 10^{-6} \text{ m})(1.6) = 3.2 \times 10^{-6} \end{aligned}$$

- (ii) The wavelength in the medium is λ_m given by

$$\lambda_m = \frac{\lambda_0}{n} = \frac{780 \text{ nm}}{1.6} = 487.5 \text{ nm}$$

where λ_0 is the free space (vacuum) wavelength.

Question 2. (Marks 15)

(a) Consider a Young's double slit apparatus in which the centre-to-centre slit spacing is 0.3mm and the slits-to-screen distance is 0.8m. Two wavelengths of light λ_1, λ_2 , illuminate the slits simultaneously, where $\lambda_1 = 500 \text{ nm}$ and $\lambda_2 = 600 \text{ nm}$, producing two interference patterns on the screen. Find the separation (distance) on the screen between the two third-order interference patterns produced by λ_1, λ_2 .

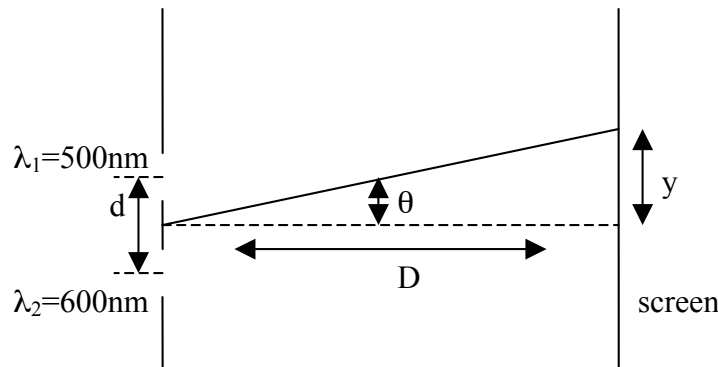
Solution:

This question involves interference only (no diffraction)

Maxima (bright fringes) at $d \sin \theta = m\lambda$

Minima (dark fringes) at $d \sin \theta = (m + \frac{1}{2})\lambda$

$m = \text{order of interference} = 0, 1, 2, 3, \dots$



$$d = 0.3 \text{ mm} = 3 \times 10^{-4} \text{ m}, D = 0.8 \text{ m}$$

3rd Order fringes $\Rightarrow m = 3$.

$$\sin \theta_1 = \frac{3 * 500 * 10^{-9}}{3 * 10^{-4}} = 5.0 * 10^{-3}$$

$$\sin \theta_2 = \frac{3 * 600 * 10^{-9}}{3 * 10^{-4}} = 6.0 * 10^{-3}$$

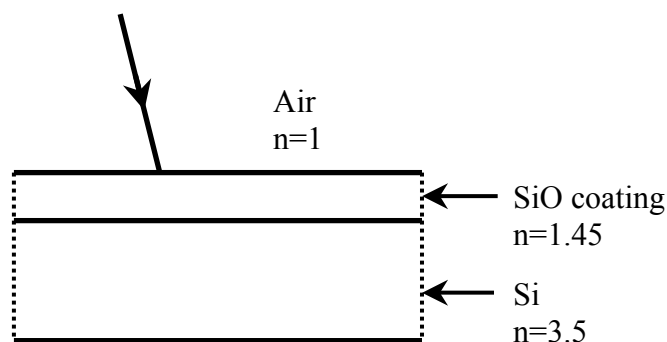
$$\tan \theta = \frac{y}{D}, D = 0.8 \text{ m}$$

for a small angle $\tan \theta \approx \sin \theta$

$$\therefore y_2 - y_1 = D(\sin \theta_2 - \sin \theta_1) = 8.0 * 10^{-4} \text{ m}$$

(b) To maximize collection efficiency by minimizing reflective losses, the surface of silicon (Si) solar cells can be coated with a thin film of silicon monoxide (SiO).

- (i) On the diagram below, representing a SiO coated Si solar cell, sketch in transmitted and reflected rays, indicating on the sketch all phase changes occurring at the air-SiO and SiO-Si interfaces.

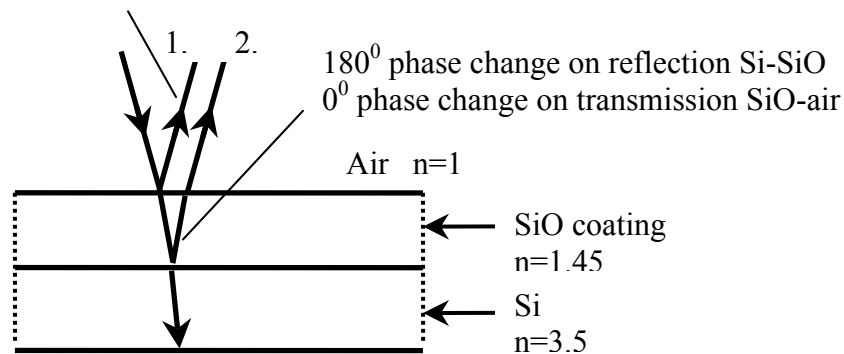


- (ii) For the SiO coated Si solar cell, calculate the minimum film thickness required to minimize reflection losses of solar radiation of wavelength $\lambda = 550 \text{ nm}$.

(Refractive indices: Silicon cell: $n_{Si} = 3.5$; Coating, $n_{SiO} = 1.45$)

Solution:

- (i) 180° phase change on reflection SiO-air
 0° phase change on transmission air-SiO



- (ii) Minimum reflection occurs when exiting rays 1. and 2. interfere destructively.

Rays 1. and 2. *both* undergo a 180° phase change upon reflection, i.e. no *net* change on reflection.

A phase change of ray 2. w.r.t. ray 1. occurs in transmission a distance $2t$ within the SiO, where t is the SiO film thickness.

$$\therefore 2nt = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4n} = \frac{550 \times 10^{-9}}{4(1.45)} = 9.48 \times 10^{-8} \text{ m} \approx 95 \text{ nm}$$

Question 3 (Marks 10)

A student attends a physics lecture with a camera to record the notes the lecturer has projected on to the theatre's screen. The camera has a lens diameter 2mm. The lecturer has written the notes in blue ink ($\lambda_{blue} = 450 \text{ nm}$) and each character (letter or symbol) can be considered to be a 3mm diameter circle on the screen.

- (i) Will the camera resolve individual characters on the screen if the student sits at the back of the theatre, at a distance of 25 m from the screen? If not,
- (ii) what is the minimum distance the camera must be from the screen such that individual characters are just resolvable?

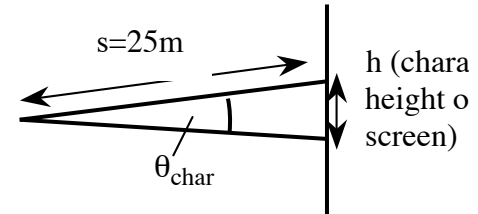
Note: a plain yes/no answer will obtain no marks; the principle and argument used to show resolvability must be given, with a simple sketch if appropriate, along with all working in your calculation.

Solution:

- (a) The camera has lens diameter $D = 2\text{mm}$. The wavelength of light from the screen is $\lambda = 450\text{nm}$. Characters on the screen have height $h = 3\text{mm}$.

- (i) If the student is 25 m from the screen we have:

$$\theta_{\text{char}} = \frac{h}{s} = \frac{3 \times 10^{-3} \text{ m}}{25 \text{ m}} = 1.2 \times 10^{-4} \text{ rad}$$



The Rayleigh Criterion gives

$$\theta_r = 1.22 \frac{\lambda}{D} = 1.22 \frac{450 \times 10^{-9}}{2 \times 10^{-3}} = 2.7 \times 10^{-4} \text{ rad}$$

$\theta_r = 2.7 \times 10^{-4} \text{ rad}$ is the angle subtended by the character on the screen at the viewer's position which allows resolution of individual characters by the camera lens, when resolution is limited by diffraction. At 25 m the character height is too small to allow resolution.

- (ii) For characters to be just resolvable by the camera lens we must have

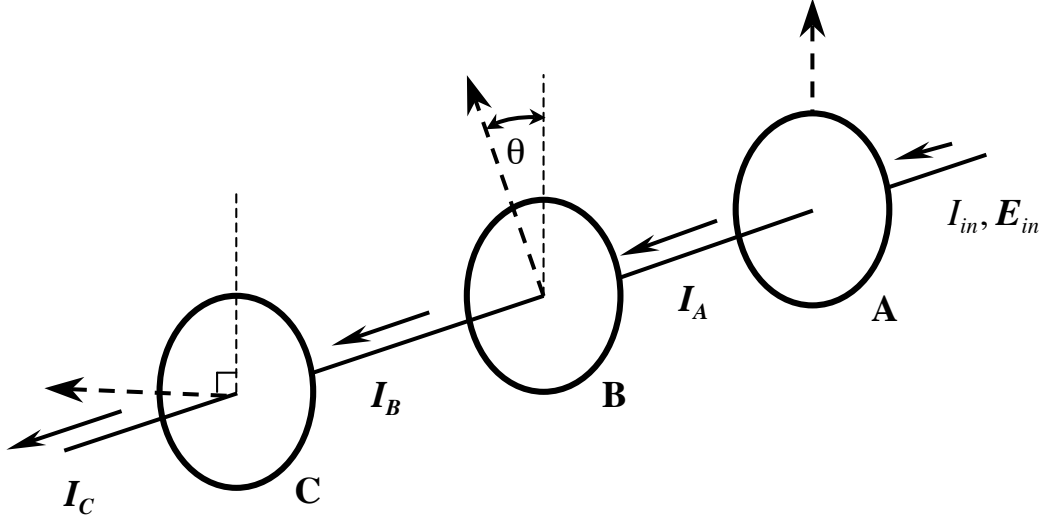
$$\theta_{\text{char}} = \theta_r = 2.7 \times 10^{-4} \text{ rad}$$

so the student must sit at, or closer than,

$$s = \frac{h}{\theta_r} = \frac{3 \times 10^{-3}}{2.7 \times 10^{-4}} = 11.1 \text{ m}$$

Question 4 (Marks 10)

Three ideal polarizing sheets are arranged as shown below. Unpolarised light of intensity I_{in} is incident upon polarising sheet A. The polarisation axes of the sheets are indicated by the broken arrows. Sheets A and C are arranged as shown with their axes of polarization at 90° , sheet C is rotated such that its polarization axis is at angle θ to the vertical (and therefore also at θ° to the polarization axis of sheet A).



- (i) What is the intensity of light transmitted by sheet A?
- (ii) If $\theta = 45^\circ$, find the intensity of light transmitted by each of the three sheets and therefore that transmitted by the system of sheets A, B and C.
- (iii) If $\theta = 30^\circ$, find the intensity of light transmitted by the system.

Solution:

- (i) What is the intensity of light transmitted by sheet A?

When polariser A absorbs one of the components (e.g. the z-component, see diagram above) half the incident intensity is removed. Therefore,

$$I_A = \frac{1}{2} I_{in}$$

- (ii) If $\theta = 45^\circ$, find the intensity of light transmitted by each of the three sheets and therefore that transmitted by the system of sheets A, B and C.

$$I_A = I_{in}/2 \text{ and using Malus' law,}$$

$$I_B = I_A \cos^2 \theta = I_A \cos^2(45^\circ) = I_A/2 = I_{in}/4,$$

$$\therefore I_C = (I_{in}/4) \cdot \cos^2(45^\circ) = I_{in}/8$$

Question 5 (11 Marks)

(a) Find the de Broglie wavelength of a 25kV electron.

Solution:

$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.6 \times 10^{-19})25 \times 10^3}{9.11 \times 10^{-31}}} = 9.37 \times 10^7 \text{ ms}^{-1}$$

(non-relativistic)

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(9.37 \times 10^7)} = 7.77 \times 10^{-12} \text{ m} \quad (= 7.77 \text{ pm})$$

(b) The lifetime of the unstable hydrogen $n = 2$ state is approximately 10ns. Using Heisenberg's Uncertainty Principle determine the number of significant figures that may be used to express its energy.

Solution:

Heisenberg: $\Delta E \Delta t = h/2\pi$.

The hydrogen energy levels are

$$E_n = -\left(\frac{me^4}{8\epsilon_0^2 h^2}\right) \frac{1}{n^2}$$

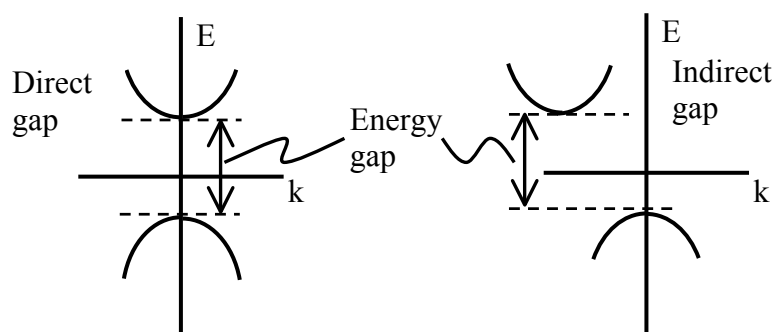
For $n = 2$ this gives $E = -3.4 \text{ eV}$

$$\Delta E/E = \frac{h}{2\pi \Delta t \cdot E} \approx 10^{-9} \text{ on substitution } E = -3.4 \text{ eV}, t = 10 \text{ ns} \quad 9 \text{ sig figs only}$$

(c) Provide a simple *labelled* sketch showing the fundamental difference between a direct and an indirect gap semiconductor. Name one semiconductor material of each type.

Solution:

In a direct gap semiconductor the minimum in conduction band is directly above maximum in valence band (at the same value of wavevector k). In an indirect gap semiconductor this is not the case:



Names of direct gap and indirect gap semiconductors. The obvious examples are (although students may cite several others):

Silicon – indirect gap

Gallium Arsenide (GaAs) – direct gap