SOLUTIONS 1231 T1

Q1. SHM Vibrating Strip

(a)(i) For SHM,

$$y = A\sin(wt + f)$$

for amplitude A and angular frequency w. Set $f = 0$.

(ii) The velocity is given by

$$v = \frac{dy}{dx} = wA\cos wt$$

The maximum speed $|v_m|$ occurs when cos = 1,

$$\therefore\ |v_m| = wA \text{ with } w = 2pu,\ n = 5Hz$$

$$|v_m| = 2p.5.(10.10^{-3})ms^{-1}$$

$$= 0.314\,ms^{-1}$$

The acceleration *a* in SHM is given by

$$a = \frac{d^2y}{dt^2} = -w^2 A\sin wt$$

The maximum value of the acceleration occurs when sin=1 with magnitude

$$|a_m| = w^2 y_m = w^2 A$$

$$\therefore\ |a_m| = w^2 A = (2pu)^2 A$$

$$= (2p.5)^2.10^{-2}\,ms^{-2} = 9.87ms^{-2}$$

(b) Bead mass = 2 g, SHM frequency = 3 Hz.

The acceleration (from part (a)) is

$$|a_m| = w^2 A = (2pu)^2 A$$

The downward force on the bead due to gravity, F_g, is

$$F_g = mg = (2x10^{-3}).9.8 = 0.0196\ N$$

The bead will begin to lose contact when $|a_m| \geq F_g$, or

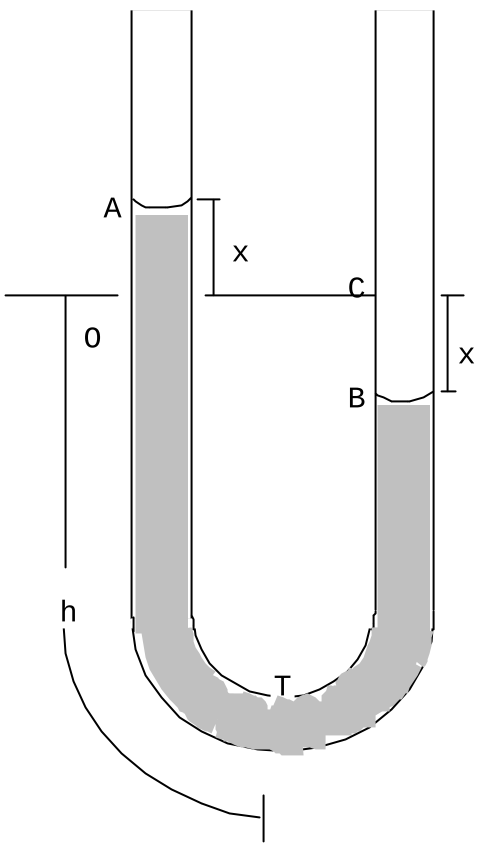
$$(2pn)^2 A \geq 0.0196$$

$$A \geq \frac{0.0196}{(2p.3)^2}$$

$$\underline{A \geq 5.5x10^{-5} m = 0.055\text{mm}}$$

2. U-tube oscillations.

The required diagram is:



Where x is the displacement from equilibrium in either arm of the U-tube, and 2h is the total column length of liquid.

When liquid is displaced by x, l.h.s. moves O$\rightarrow$A, r.h.s. moves C $\rightarrow$B

Excess pressure on whole liquid = excess height x density x g $= 2xrg$

Since pressure = force per unit area,

force on liquid = pressure x cross-sectional area of tube

$$= 2xrgA$$

(r =density of liquid, A=cross-sectional area of tube)

Excess pressure causes (restoring) force which accelerates liquid $\Rightarrow$Newton's 2nd Law: F=ma.

Total mass of liquid in tube = $2hAr$ (2h is total length of liquid column)

So F=ma becomes:

$$-(2x\,rgA) = (2hAr)(a)$$

force mass x acceleration

(minus sign indicates acceleration directed towards equilibrium position)

Rearranging this we have: $a = -\frac{g}{h}x = -w^2 x$

where $w^2 x$ is the acceleration in SHM with frequency w and

$$w = \sqrt{\frac{g}{h}}$$

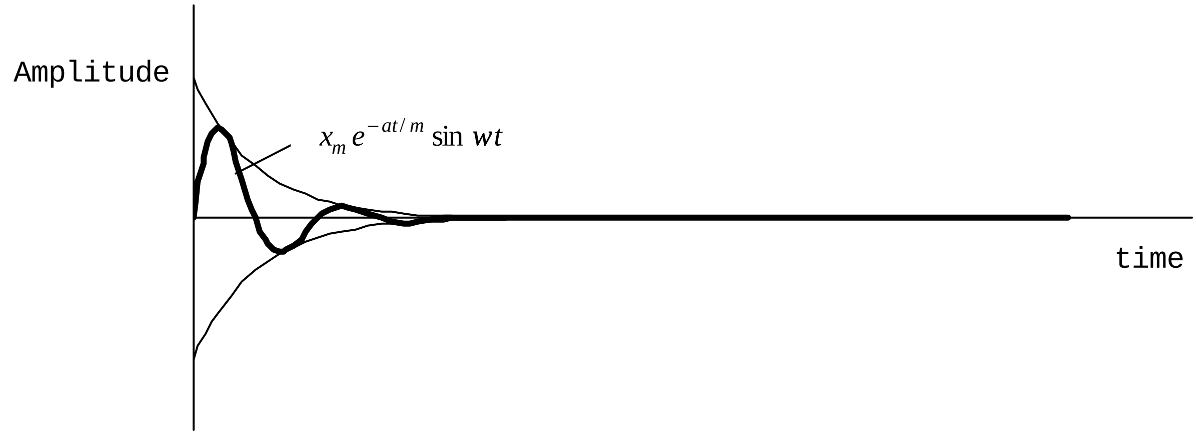
The period of oscillation is $T = \frac{2p}{w} = 2p\sqrt{\frac{h}{g}}$

(b) For the three different liquids in the U-tube the SHM is ***damped*** to differing degrees. The motion can be represented graphically by an exponentially decaying sinusoid.

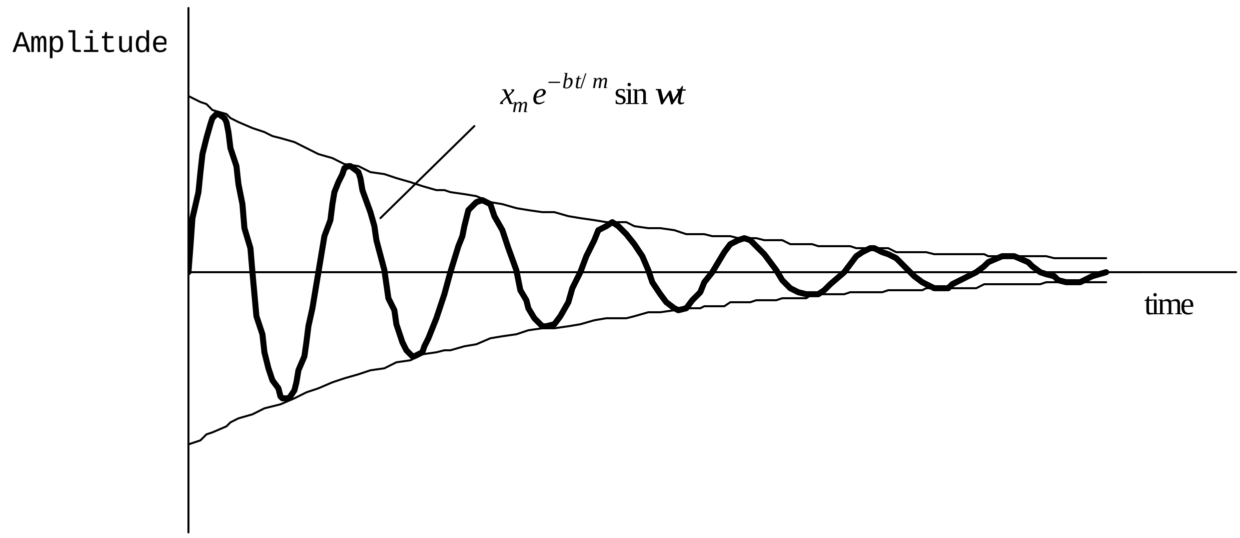
In the diagrams below, the decay envelope has the form $x \sim e^{-t}$ with m=mass and damping constants *a,b* where $a > b$ indicates heavier damping in case (i).

[case (iii), undamped SHM is not generally realised in practice but could be observed in special circumstances, e.g. superfluid oscillations in a U-tube or oscillations in a very high Q system – these were mentioned in lectures but students not expected to know it]

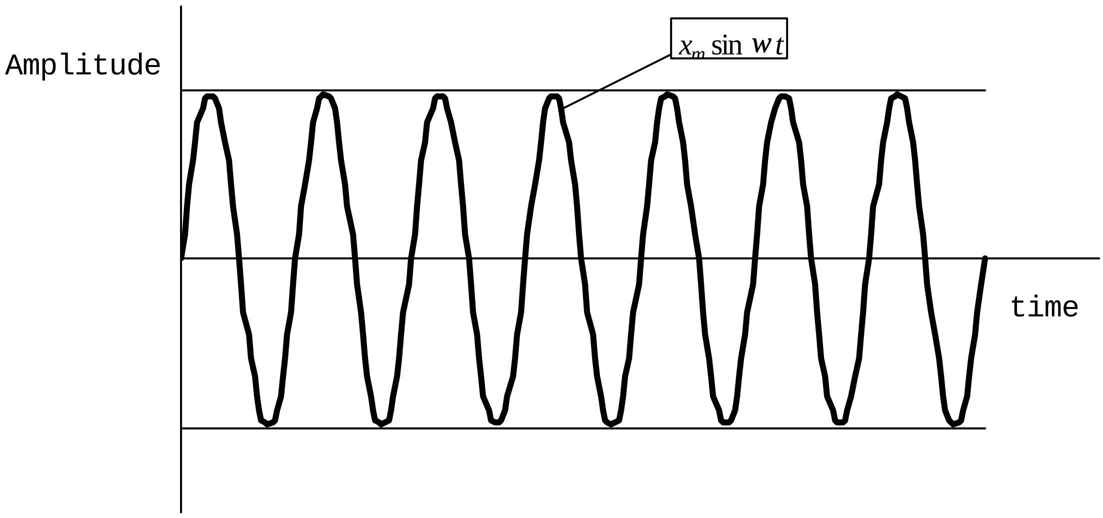
(i) Heavy damping (very viscous liquid)



(ii) Light damping (moderate viscosity)



(iii) Undamped motion (zero viscosity liquid)



3. Standing waves on string

(i) The two waves given are $y_1 = 0.20\sin(2.0x - 4.0t)$ and $y_2 = 0.20\sin(2.0x + 4.0t)$ and are of the form:

$$y = y_m \sin(kx \pm \omega t)$$

so that $k = 2.0m^{-1}$ and $\omega = 4.0s^{-1}$ by inspection.

Using the identity $\sin A + \sin B = 2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$ where A and B represent the two wave functions given, the standing wave is y_{1+2} given by

$$y_{1+2} = 2y_m \sin kx \cos \omega t = 0.40\sin(2.0x)\cos(4.0t)$$

(ii) At position $x = 0.45m$

$$y = 0.40\sin(0.90)\cos(4.0t) = 0.31\cos(4.0t)$$

$\therefore$ maximum amplitude with value y=0.31m occurs when cos(4.0t)=1

(iii) For the standing wave pattern

$$y_{1+2} = 0.40\sin(2.0x)\cos(4.0t)$$

we will have **nodes** at both ends of the string. For such a string fixed at both ends, nodes are also located at positions $x = n\frac{\lambda}{2}$, so that

$$\frac{\lambda}{2} = \frac{1}{2}\frac{2\pi}{k} = \frac{\pi}{2.0}m = 1.57m$$

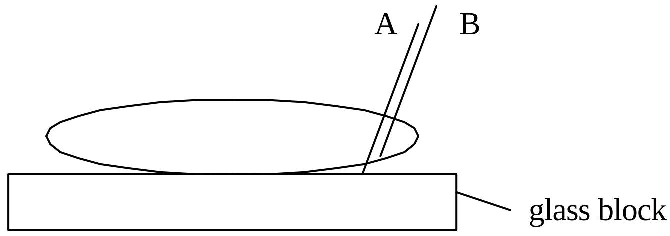
A standing wave will result when the other end of the string is fixed at *x* position

$$x = n(1.57m) = 1.57m, 3.14m, \ldots\ldots \quad (n=1,2,3\ldots)$$

(iv) Nodes will occur at x=0,1.57m, 3.14m…… The maximum amplitude is y=0.40m located at positions mid-way between nodes. Assuming the string is fixed at x=0 and x=1.57 the maximum amplitude 0.40m is located at x=0.785m.

4. Newton's rings.

The geometry of this arrangement is:



Ray A: no phase change on *reflection* from glass block surface into air

Ray B: π phase change on *reflection* from lens' lower surface

Transmission of rays: no phase change

(a) The condition for maxima is

$$2d = (m + \frac{1}{2})\lambda \qquad m = 0,1,2,3,\ldots$$

and for the 5th bright ring (maximum)

$$2d = (4 + \frac{1}{2})\lambda \qquad \text{(note: m=0 is the first)}$$

$$d = \frac{9}{4}\lambda = \frac{9}{4}(546\text{x}10^{-9}) = 1228.5\text{x}10^{-9}\,m$$

$$\underline{d = 1228.5nm}$$

(b) Immersion in the transparent fluid changes the *optical path length* between lens' lower surface and glass block

$$\underbrace{2.d}_{\text{in air}} \Rightarrow \underbrace{2.n.d}_{\text{in fluid}} \qquad \text{(optical path length)}$$

where n is the refractive index of the fluid.

In air, dark rings occur at

$$2d = m\lambda \qquad \text{(m=0,1,2,…)}$$

So, in air, the 3rd dark ring is at

$$2d = 3\lambda$$

$$d = \frac{3}{2}\lambda \qquad \text{(in air)}$$

If the 5th bright fringe now occupies the position (when in the fluid) that the 3rd dark fringe had (in air), we have

$$(4+\frac{1}{2})\lambda = 2nd = 2n(\frac{3}{2}\lambda)$$

optical path length

$$4.5 = 3n$$

$$\underline{n = 1.5}$$

5. Two Slit Interference

(a) Linear separation of fringes on the screen:

Maxima are observed for $d\sin\theta = m\lambda$

Where d is slit separation, m is integer. For two *adjacent* fringes we have

$$d\sin\theta_1 = m\lambda$$

$$d\sin\theta_2 = (m+1)\lambda$$

where θ_1, θ_2 are the angular positions of the adjacent fringes. Since the slit-screen separation is 1m we have to a good approximation $\sin\theta \cong \theta$

$$\therefore \qquad \Delta q = q_2 - q_1 = \frac{l}{d} = \frac{600\,\mathrm{x}10^{-9}}{0.5\mathrm{x}10^{-3}} = 1.2\mathrm{x}10^{-3} rad$$

The *linear* distance between fringes on the screen will be

$$d = L\Delta q = (1m)\mathrm{x}(1.2x10^{-3} rad) = 1.2\,mm$$

(b) The intensity pattern on the screen is the product of the *interference* effect modulated by the *diffraction* 'envelope'.

Diffraction minima occur according to the condition

$$a \sin q = n l \qquad\qquad (n \text{ integer})$$

where *a* is the slit width. For *n*=1,

$$q = \frac{l}{d} = \frac{600\,\mathrm{x}10^{-9}}{0.1\mathrm{x}10^{-3}} = 6\,\mathrm{x}10^{-3} rad$$

The 5th interference fringe in this pattern has zero intensity – it is 'modulated' to zero by the diffraction envelope. The pattern looks like:

(c) The intensity envelope arises because diffraction at each slit modulates the 'strength' (intensity) of the interference fringes.

Using a phasor diagram:

The slit pattern has N sub-rays each differing in phase f by

$$f = \frac{2pa}{l}\sin q$$

for slit width a, angular position on screen q .

Referring to the diagram,

$$f = \frac{E_m}{R} \text{ and } E_q = 2R\sin\frac{f}{2} = \frac{Em}{\frac{f}{2}}\left\{\sin\frac{f}{2}\right\} = Em\left\{\frac{\sin f/2}{f/2}\right\}^2$$

and the intensity is

$$I_q = I_m\left\{\frac{\sin f/2}{f/2}\right\}^2$$

The intensity of the 3rd fringe relative to the central maximum is

$$I_q = I_m \cos^2 b \left\{ \frac{\sin f/2}{f/2} \right\}$$

$$\text{where } b = \frac{pd}{l} \sin q = 3p$$

$(d \sin q = 3l \Rightarrow \sin q = 3.6\text{x}10^{-3})$

$$f/2 = \frac{pa}{l} \sin q = \frac{pa}{l}(3.6\text{x}10^{-3}) = 0.6p$$

$$\therefore \qquad I_3 = I_m \cos^2(3p) \left[\frac{\sin 0.6p}{0.6p} \right]^2 = 0.255\, I_m$$

(d) If there are four (rather than two) slits of equal width, the width of the interference fringes is **reduced** according to

$$\Delta q = \frac{l}{Nd}$$

where Δq is the angular width of interference fringes, λ is the wavelength of illuminating light and N is the number of slits.

In this case, doubling the number of (equivalent) slits *halves* the fringe widths to new value 0.3x10^{-3} rad.