SOLUTION to T1R S2 2000

Question 1 (16 Marks)

(a) The velocity is $v = \frac{dx}{dt} = 1.60(1.30)\cos(1.30t - 0.75)cm/s$ and the acceleration is

$$a = \frac{dv}{dt} = -1.60(1.30)^2 \sin(1.30t - 0.75)cm^2/s$$

$$= -(1.30)^2 y$$

At t=0 s

Displacement: $y = 1.60\sin(-0.75) = -1.60\sin 43^\circ = -1.09cm$
Velocity: $v = 2.08\cos(-43^\circ) = 1.52\,cm/s$
Acceleration $a = -(1.30)^2(1.09) = 1.84cm^2/s$

At t=0.60 s

Displacement: $y = 1.60\sin(0.03) \approx 1.60(0.03) = 0.048\,cm$
Velocity: $v = 2.08\cos(0.03) \approx 2.08cm/s$
Acceleration $a = -(1.30)^2(0.048) = -0.081cm^2/s$

(b) A torsional pendulum has a period given by

$$T = 2p\sqrt{\frac{I}{K}}$$

In the question, a torque of 5Nm produces a deflection of 12° which gives a spring constant, *K*, given by

$$K = \frac{\text{external torque}}{\text{angular displacement}} = \frac{5Nm}{12^\circ\left(\frac{2p}{360}\right)} = 23.9\text{Nm/rad}$$

$$I = \left(\frac{T}{2p}\right)^2 K = \left(\frac{0.5s}{2p}\right)^2 (23.9) = 0.151\,kgm^2$$

Question 2 (18 Marks)

Let source 1 emit waves in the positive x-direction such that

$$y_1 = y_{1m} \sin\ 2pn_1(t - \frac{x_1}{v})$$

and source 2 emit in the negative x-direction:

$$y_2 = y_{2m} \sin\ 2pn_2(t + \frac{x_2}{v})$$

We are told v=3m/s. x_1, x_2 are measured from source 1, source 2 respectively.

Equating y_1 at $x_1 = 0$ to y_{s1} and y_2 at $x_2 = 0$ to y_{s2} we find

$$y_{1m} = 0.06m \quad n_1 = n_2 = 0.5Hz \text{ and } y_{2m} = 0.02\, m$$

Superposition of the 2 waves at $x_1 = 12m$ and $x_2 = -8m$ gives

$$y = y_1 + y_2 = 0.06 \sin \pi(t - \frac{12}{3}) + 0.02 \sin \pi(t - \frac{8}{3})$$

$$= 0.06 \sin \pi t + 0.02 \sin(\pi t - \frac{2\pi}{3})$$

$$= 0.06 \sin \pi t + 0.02 (\sin \pi t \cos \frac{2\pi}{3} - \cos \pi t \sin \frac{2\pi}{3})$$

$$= 0.06 \sin \pi t + 0.02[\sin \pi t(-\frac{1}{2}) - \cos \pi t(\frac{\sqrt{3}}{2})]$$

$$= 0.05 \sin \pi t - 0.0173 \cos \pi t$$

(b) The sound (pressure) wave is given as

$$p = 1.5 \sin\{[\frac{2p}{l}](x - 330\, t)\}$$

A travelling wave equation in standard form is

$$y = y_m \sin\left[2\pi n(t - \frac{x}{v})\right]$$

with amplitude y_m, frequency n and velocity v. Rewriting the given sound wave in standard form,

$$p = -1.5\sin\left[2\pi\frac{330}{\lambda}(t - \frac{x}{330})\right]$$

whence,

(i) $v = 330 m/s$,

(ii) $v = n\lambda$ so $n = \frac{330}{2} = 165\,Hz$

(iii) amplitude $p_0 = 1.5 Pa$

(iv) at x=1/6m and t=0, by subsituting in,

$$p = -1.5\sin\left[2\pi\frac{330}{2}(0 - \frac{1/6}{330})\right] = 1.5\sin\frac{\pi}{6}$$
$$\underline{= 0.75 Pa}$$

Question **3 (13 Marks)**

The m^th bright fringe due to λ_1 and the k^th bright fringe due to λ_2 are at positions

$$y_m = \frac{mD\lambda_1}{d} \quad \text{and} \quad y'_m = \frac{kD\lambda_2}{d}$$

For a bright fringe from each wavelength at the *same* location,

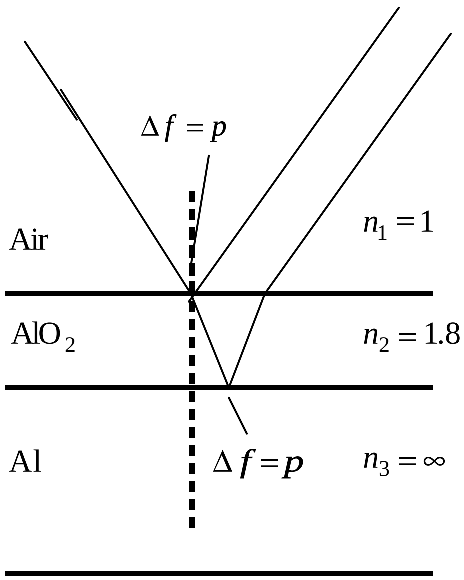
$$\frac{m}{k} = \frac{\lambda_2}{\lambda_1} = \frac{900}{750} = \frac{6}{5}$$

and $$y_6 = y'_5 = \frac{(6)(2)(750\text{x}10^{-9})}{2\text{x}10^{-3}} = 4.5\text{x}10^{-3} m$$

$= 4.5mm$

Question 4 (19 Marks)

The diagram required is



We require wavelengths, l_1 in air (with refractive index) which fall in the visible range (400nm to 700nm) and interfere firstly, constructively

$$l_1 = \frac{2(n_2/n_1)t}{m+1} = \frac{(2)(1.80/1)(250\,\text{x}10^{-9})}{m+1} \qquad \text{and } m=0,1,2,\ldots$$

or

$$l_1 = \frac{900\ \text{x}10^{-9}}{m+1} = \frac{900\,nm}{m+1} \qquad (\textit{constructive}\text{ interference})$$

for which m=1 gives $l_1 = 450\,nm$ (blue/indigo), all other wavelengths are outside the visible range, and destructively,

$$l_1 = \frac{2(n_2/n_1)t}{m+\frac{1}{2}} = \frac{(900nm)}{m+\frac{1}{2}} \qquad (\textit{destructive}\text{ interference})$$

where m=1 giving $l_1 = 600\ nm$ (orange), is the only visible wavelength.

The medals will appear blue/violet, since light reflected at the red/orange end of the spectrum will be attenuated and light from the blue/violet end of the spectrum will be most strongly reflected.

Question 5 (14 Marks)

(a) (i) The polariser reduces the incident intensity I_0 to $I = \frac{I_0}{2}$. The analyser will transmit an intensity I' given by

$$I' = I \cos^2 q = \frac{I_0}{2} \cos^2 30^\circ = 0.375 I_0$$

(ii) The polariser will transmit an intensity

$$I = I_0 \cos^2 q = I_0 \cos^2 30^\circ = 0.75 I_0$$

which is polarised at 30 degrees to the analyser's axis, so that the final transmitted intensity is

$$I_f = I \cos^2 q = 0.75 I_0 \cos^2 30^\circ = 0.563 I_0$$

(b) Quarter wave plate: For plate thickness l, the optical paths lengths of the ordinary and extraordinary rays are $ln_\perp$ and $ln_{||}$ respectively.

The two rays must emerge with a 90º phase difference so the optical paths must differ by $(k + \frac{1}{4}) l_0$ with k=0,1,2,3…

The minimum thickness is given by

$$\frac{l_0}{4} = l(n_\perp - n_{||})$$

so that

$$l = \frac{l_0}{4(n_\perp - n_{||})} = \frac{589\text{x}10^{-9}}{4(1.732 - 1.456)} = 534\text{x}10^{-9} m = 534 nm$$