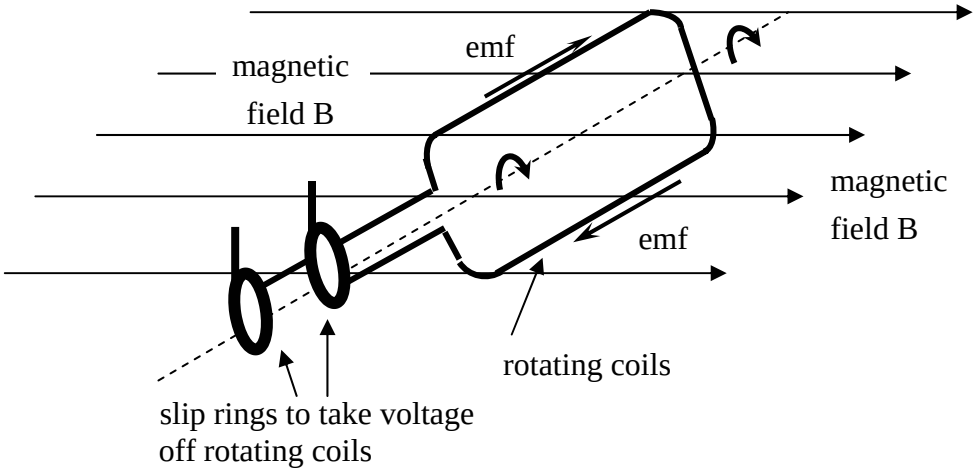
Question 6 (Marks)

(a) AC generator:



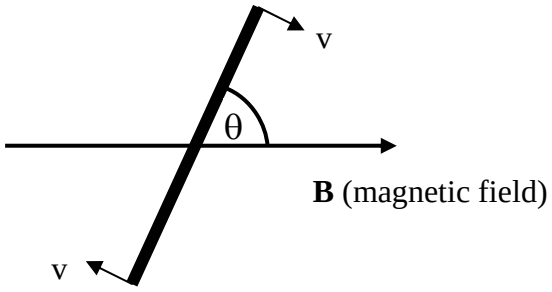
Uniform external magnetic field B tesla applied perpendicular to the long axis of the coils (shown as an open rectangle above). The coils consist of N (many) turns of copper wire.
Conduction electrons in the wire experience a force in the magnetic field B given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

This force on the conduction electrons generates an emf in the coils in the direction indicated in the figure. The emf generated in one of the long sides of the coil has magnitude

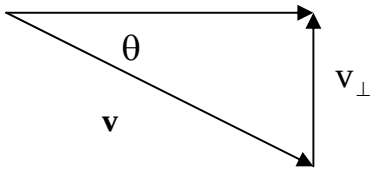
$$\text{emf} = \varepsilon = BLv_{\perp}$$

where L is the length of one of the long sides of the coil and $v_{\perp}$ is the perpendicular component of the velocity in the magnetic field.
The coils are rotating in the magnetic field such that end-on they look like:



So that

$$v_\perp = v\sin\theta$$



and,

$$\varepsilon = BLv_\perp$$

$$= 2BLv\sin\theta \quad \text{(for a single-turn coil).}$$

If the generating coil has N turns we will have

$$\varepsilon = 2NBLv\sin\theta$$

Therefore, as the coils rotate we generate a sinusoidally varying emf.
The tangential speed of the coil v is related to the angular speed ω by

$$v = r\omega$$

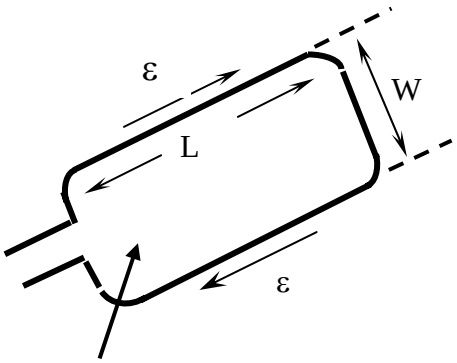
Here we have

$$r = W/2$$

and

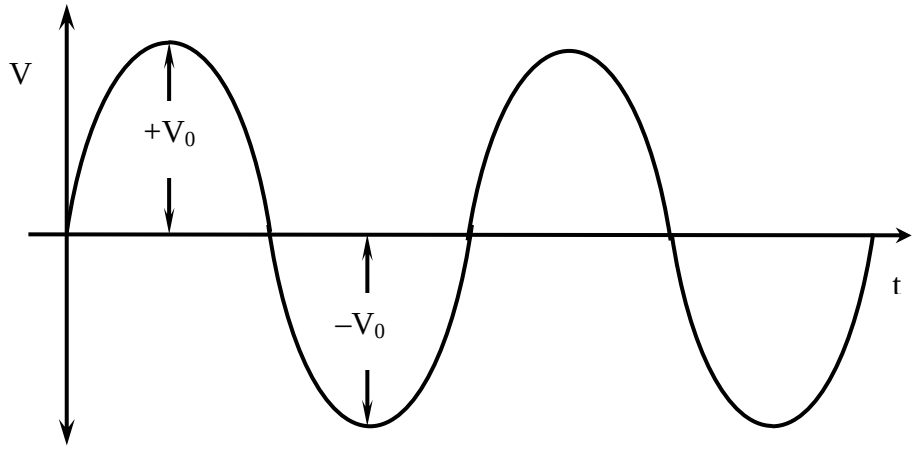
$$\theta = \omega t$$

so that, $\varepsilon = NAB\omega\sin\omega t$



area A = LxW

The voltage output, V(t) of this generator across a load will look like:



(b) $\varepsilon = NAB\omega\sin\omega t$

$N = 400$, $A = (6x10^{-2})(4x10^{-2}) = 2.4x10^{-3}\ m^2$

$$B = \mu H = \mu_0 H = (4\pi x10^{-7})(100) = 400\pi x10^{-7}\ T\ T$$

$$\omega = 2\pi f = 2\pi \frac{600}{60} = 20\pi\ \text{rads}^{-1}$$

$$\varepsilon = (400)(2.4x10^{-3})(400\pi x10^{-7})(20\pi)\sin\omega t$$

$$\varepsilon = (7.68x10^{-4})\pi^2 \sin\omega t = (7.58x10^{-3})\sin\omega t$$

and the r.m.s. value is

$$\varepsilon_{rms} = \frac{\varepsilon_{peak}}{\sqrt{2}} = \frac{7.58x10^{-3}}{\sqrt{2}} = 5.36x10^{-3}\,V = 5.36\ mV$$

(c) Faraday's law is

$$\varepsilon = -\frac{d\Phi}{dt}\ \text{(for a single turn coil) where}\ \Phi = \mathbf{B}.\mathbf{A}$$

$$A = (6x10^{-2})(4x10^{-2}) = 2.4x10^{-3}\ m^2$$

The magnetic flux is

$$\Phi = \mathbf{B}.\mathbf{A} = \mu_0\mathbf{H}.\mathbf{A} = (4\pi x10^{-7})(100)(2.4x10^{-3}).\cos\theta = (9.6x10^{-8}\pi)\cos\theta$$

where θ is angle between **H** and **A**, and

$$\theta = \omega t = 2\pi ft$$

and

$$\varepsilon = -\frac{d\Phi}{dt} = (400)(9.6x10^{-8}\pi)(20\pi)\sin(2\pi ft)$$

$$= (7.58x10^{-3})\sin 2\pi ft$$

$$|\varepsilon_{rms}| = \frac{7.58x10^{-3}}{\sqrt{2}}\ V = 5.36x10^{-3}\ V = 5.36\ mV$$