QUESTION 4 (Marks 26)

(a) (i) The drift velocity comes from $I = nAve \Rightarrow v = \frac{I}{nAe}$

The carrier concentration is

$$n = \frac{\rho N_A}{M} = \frac{(8.9x10^3)(6.02x10^{23})}{63.5x10^{-3}} = 8.43x10^{28}\,m^{-3}$$

$$v = \frac{I}{nAe} = \frac{100x10^{-3}}{(8.43x10^{28})(\pi\left(\frac{(0.2x10^{-3})^2}{4}\right)1.6x10^{-19}}$$

$$= 2.36x10^{-4}\,ms^{-1} \approx 0.2mm.s^{-1}$$

(ii) This is a surprisingly small speed when compared to magnitude of other electronic speeds and a very tiny fraction of the speed of light.

(b) (i) The circumference of the coil former is

$$2\pi r = 2\pi(5x10^{-3}) = 0.0314m.$$

The 2m length of wire will form a coil 5mm in diameter with $\frac{2.0}{0.0314} = 63.69 \approx 64$ turns in a length $(64)(0.2x10^{-3})m = 0.0128m$.

The defining equation for inductance (air core) is

$$N\Phi = Li$$

so

$$L = \frac{N\Phi}{i} = \frac{NBA}{I} \approx \frac{\mu_0 N^2 A}{l}$$

and with iron former,

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

$$L \approx \frac{(4\pi x10^{-7})(600)(64)^2 (\pi(5x10^{-3})^2 / 4)}{0.0128} = 1.49x10^{-2}\,H = 14.9mH$$

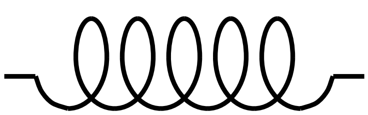
(ii) Without iron former L is just reduced by factor $1/\mu_r = 1/600$.

$$L_{air} = \frac{1.49x10^{-2}}{600}\,H = 2.48x10^{-5}\,H = 24.8\,\mu H$$

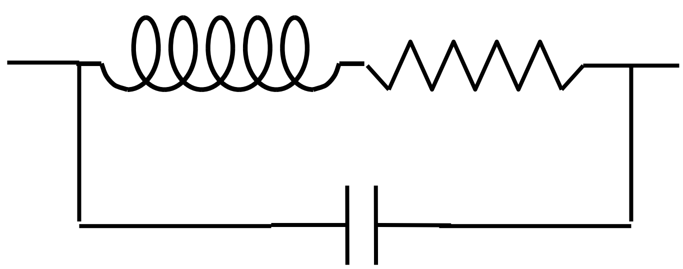
(iii) Possible differences are:

• Practical inductor has series resistance (of windings).

• Real inductor has a distributed parasitic capacitance leading to the inductor having a self resonant frequency due to the proximity of adjacent windings and the dielectric effect of insulation.



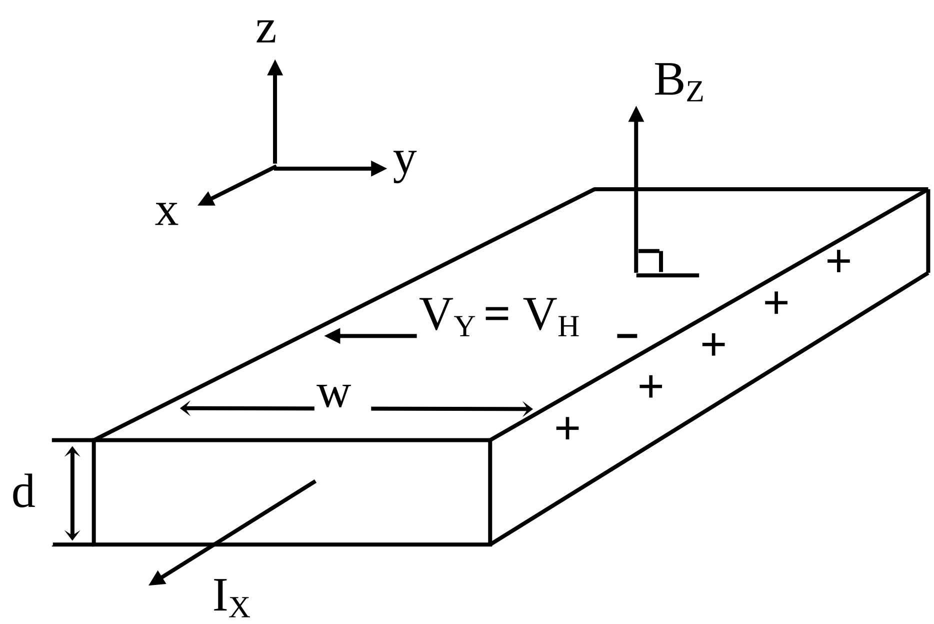
ideal inductor



practical inductor

Practical inductors are often not air-cored. The core material (powdered iron or ferrite) can lead to a number of differences to predicted ideal behaviour, e.g. magnetic losses in core can generate heating, behaviour of inductor can have complex frequency dependence etc.

(c) (i) For a Hall effect measurement, the arrangement is:




(ii) Conventional current I in +x direction, electrons move toward –x direction, this is direction of v.

Force on charge carriers is $\mathbf{F}_m$,

$$\mathbf{F}_m = q(\mathbf{v} \mathrm{x} \mathbf{B})$$

For electrons, $q = -e$; $\mathbf{F}_m$ pushes electrons in –y direction leaving net +ve charge on one of the long faces, as shown. This charge separation produces the Hall electric field and a Hall voltage, $V_H$ in direction shown.

When the Hall voltage is established the magnetic force on the electrons is balanced by the electric force due to the Hall field so that

$$eE_y = -B_z e v_x$$

From $I = nAve$ we see the current density J is

$$J_x = \frac{I_x}{A} = nev_x$$

The subscripts on J, I and v give their directions. Since $J = \sigma E$ where $\sigma$ is the conductivity, we have

$$J_x = \frac{I}{A} = \sigma E_x = nev_x$$

Rearranging these we find that

$$E_y = -\frac{B_z J_x}{ne} = -\frac{B_z E_x \sigma}{ne}$$

Note that

$$E_y = \frac{V_y}{w} = \frac{V_H}{w}$$

where w is the width of the specimen in metres.

So, if the current, I, and magnetic field, B, are known, measurement of the Hall voltage $V_H$ gives us the electron concentration n:

$$n = -\frac{B_z J_x}{E_y e} = -\frac{w B_z J_x}{V_H e}$$

or

$$n = -\frac{w B_z I_x}{A V_H e}$$

where A is the cross-sectional area of the specimen.

Since A=w x d (cross-sectional area A = width w times thickness d)

we can also write, $n = -\frac{B_z I_x}{d V_H e}$

(iii) Magnitude of the Hall voltage. From above,

$$E_y = E_H = -\frac{B_z J_x}{ne}$$

and the Hall voltage is

$$V_y = V_H = -\frac{B_z I_x}{dne}$$

where d is the thickness of the strip, $d = 0.01\text{mm} = 1.0\text{x}10^{-5}\,\text{m}$

$$V_H = -\frac{B_z I_x}{dne} = \frac{(0.1\text{T})(1.5\text{A})}{(1x10^{-5}\,\text{m})(8.5x10^{28})(1.6x10^{-19})} = 1.10x10^{-6}\,\text{V} = 1.1\mu\text{V}$$

$$|\varepsilon_{rms}| = \frac{7.58\text{x}10^{-3}}{\sqrt{2}}\text{V} = 5.36\text{x}10^{-3}\,\text{V} = 5.36\text{mV}$$