THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2009

PHYS3230

ELECTROMAGENTISM

Time Allowed – 2 hours

Total number of questions - 3

Answer ALL questions

All questions ARE NOT of equal value

Candidates must supply their own, university approved calculator.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Candidates may keep this paper.

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Electromagnetism Phys 3230

Exam 2009

All questions should be addressed. If one is not certain in maths, one should try to present explanations in words.

1. **Maxwell's equations** (40%)
   a. Write down Maxwell's equations in vacuum in conventional differential form.
   b. Write down the current conservation law and verify that it complies with the Maxwell's equations.
   c. Rewrite the Maxwell's equations in an integral form using the Gauss's and Stokes' theorems.
   d. Consider a long and wide plane (infinite in y and z directions), which has finite thickness d (in the x-direction), call it a "wall", see its sketch in Fig.1. Presume that there are charges, which are homogeneously distributed with density ρ inside the wall and which move with the velocity $\mathbf{v}$ along the y-axes,.

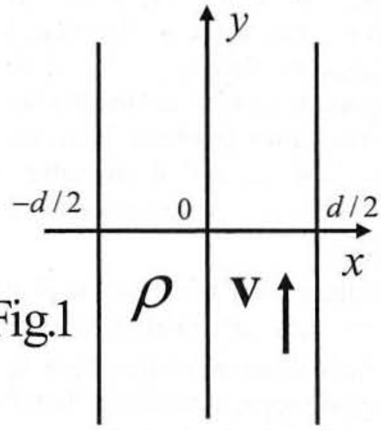

Fig.1

   - Find the electric field inside and outside the wall. How the field depends on the velocity ?
   - Find the magnetic field inside and outside the wall.

   (Hint: it is convenient to use the integral form of the Maxwell's equations)

   e. Explain *very* briefly the physical meaning of

$$u_{em} = \frac{\varepsilon_0 \mathbf{E}^2}{2} + \frac{\mathbf{B}^2}{2\mu_0}$$

$$\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0}$$

$$T_{ij} = \varepsilon_0 \left(E_i E_j - \frac{1}{2} E^2 \delta_{ij} \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} B^2 \delta_{ij} \right)$$

f. Write down Maxwell's equations in matter, which is characterized by the dielectric constant $\varepsilon(\omega)$ and magnetic constant $\mu(\omega)$, derivation is not necessary.

(Hint: remember the rule $\frac{\partial}{\partial t} \to -i\omega$, which is valid for the Fourier components.)

Explain which restrictions on the frequency should be satisfied to make these equations valid.

g. Derive Maxwell's equations in metals, assuming that a metal is characterized by the conductivity σ. Presume that the conductivity is the main effect, while atomic polarizablity and magnetized can be neglected. Present explicitly an expressions for $\varepsilon(\omega)$ for metals. Identify those ranges of frequencies where the metal is "good" and "poor".

2. Electromagnetic waves (30%)

a. Derive the wave equation for the magnetic field $\mathbf{B}$ in the vacuum from the Maxwell's equations, and by analogy, state the wave equation for the electric field $\mathbf{E}$.

b. Consider a monochromatic wave that is linearly polarized along the axes y, has a frequency ω and propagates in the x direction. Write down its electric and magnetic fields, verify that they satisfy the wave equation, as well as Gauss's law.

c. Write down (without derivation) the wave equations for the fields $\mathbf{E}$ and $\mathbf{B}$ in matter, which is characterized by the dielectric constant $\varepsilon(\omega)$ and the magnetic constant $\mu(\omega)$.

d. Explain what is called the phase and group velocity, and how they are related to $\varepsilon(\omega), \mu(\omega)$. State conditions, which these velocities have to satisfy to ensure that the signal propagates slower than the velocity of light.

3. Radiation phenomena (30%)

a. Present expressions for the intensity $I = \frac{dE}{dt}$ and angular distribution $\frac{dI}{d\Omega}$ of the E1 dipole radiation produced by a dipole moment $d(t)$.

(Hint: if you do not remember an exact form for some necessary expression here and below, you can try to derive them using the conventional dimensional counting and common sense. Numerical coefficients, which cannot be recovered this way, are of lesser importance.)

b. Consider a charge q, which moves with acceleration a during a period of time t. Find the total energy W radiated during this process, presuming that the radiation has the

E1, dipole nature. Present condition that should be satisfied to justify that the dipole process plays a dominant role.

c. Consider a capacitor, which is made of two large square metal plates separated by a small distance b, which has the capacitance C. Assume that an oscillating voltage $V(t) = V_0 \sin \omega t$ is applied to the capacitor, V_0, ω are the amplitude and frequency of the oscillations. Find the intensity of radiation $I = \frac{dE}{dt}$ presuming that it has the dipole nature. Write down conditions, which need to be satisfied to justify a dominant role of the dipole radiation.