THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2008

PHYS3230

ELECTROMAGENTISM

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions ARE of equal value

Candidates may not bring their own calculators.

The following materials will be provided by the Enrolment and Assessment Section: Calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Candidates may keep this paper.

PPS

Electromagnetism, Phys 3230

Exam 2008

All four questions have same value 25 %, all of them need to be addressed. If maths presents a problem, give a brief description of the main idea.

1. Fields and potentials

a. Write down the Maxwell's equations for the fields **E, B**.

b. Express the fields **E**, **B** via the potentials V, **A**.

c. Present gauge transformations for the potentials, i.e. fill in the right-hand sides in Eqs.(1.1)

$$\begin{aligned} V &\to V' = ? \\ \mathbf{A} &\to \mathbf{A}' = ? \end{aligned} \tag{1.1}$$

Explain briefly, in simple physical terms what the gauge invariance means. Prove that your gauge transformations from Eqs. (1.1) satisfy gauge invariance.

d. Derive the equations for the potentials V, **A** from Maxwell's equations presuming that the Lorentz gauge condition (1.2) is satisfied

$$\frac{1}{c^2}\frac{\partial V}{\partial t} + \nabla \cdot \mathbf{A} = 0 \tag{1.2}$$

e. Rewrite the equations for the potentials in the Coulomb gauge, in which it is presumed that

$$\nabla \cdot \mathbf{A} = 0 \tag{1.3}$$

2. Conservation laws

a. Current conservation

Explain briefly what current conservation means in simple physical terms. Present the current conservation law in differential form, in terms of the density of charge ρ and the current density **j**

b. Energy conservation

- Explain *very* briefly the physical meaning of

$$u_{em} = \frac{\varepsilon_0 \mathbf{E}^2}{2} + \frac{\mathbf{B}^2}{2\mu_0}, \text{ and } \mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} \tag{2.1}$$

(Remember that **S** has two interpretations.)

- Write down the energy conservation law for the system, which consists of the electromagnetic field and charged particles.
- Now be more specific, derive the energy conservation law, i.e. derive the Pointing theorem.

c. Momentum conservation.

Explain *very* briefly the physical meaning of the stress tensor

$$\vec{T}_{ij} = \varepsilon_0\left(E_i E_j - \frac{1}{2}\delta_{ij}\mathbf{E}^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij}\mathbf{B}^2\right) \tag{2.2}$$

d. Write down the momentum conservation law for a system, which consists of the electromagnetic field and charged particles. (The derivation *is not* required.)
Hint: Remember there are two ways to look at $\vec{T}_{ij}$.

3. Electromagnetic field and electromagnetic waves in matter

a. Write down Maxwell's equations in matter
b. Derive these equations.
c. Hint: keep in mind that the polarization and magnetization vectors **P**, **M** produce the charge and current densities

$$\begin{aligned} \rho_{pol} &= -\nabla \cdot \mathbf{P} \\ \mathbf{j}_{pol} &= \frac{\partial \mathbf{P}}{\partial t} \\ \mathbf{j}_{mag} &= \nabla \times \mathbf{M} \end{aligned} \qquad (3.1)$$

d. Explain briefly in simple physical terms the physical meaning of the fields $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}$.
e. Derive the wave equations for the potentials, which govern propagation of the electromagnetic waves in the vacuum. (Hint: use Q1 d.)
f. Derive the wave equation for the fields **E**, **B** in a linear medium from Maxwell's equations. It suffices to derive only one equation, for example for **E**, and just present the other one.
g. Hint. Remember that in a linear medium
h.
$$\begin{aligned} \mathbf{D} &= \varepsilon \mathbf{E} \\ \mathbf{B} &= \mu \mathbf{H} \end{aligned} \qquad (3.2)$$
i. Present the expression for the velocity of light in a linear medium presuming that ε, μ are frequency independent. Present a restriction, which follows from this expression on the product $\varepsilon\mu$.

4. Radiation processes

Consider the Hydrogen atom. Presume the simple model, in which the electron motion is described by a one-dimensional oscillator where the amplitude of oscillations has the size of an atom, $a \approx 10^{-10} m$, and the frequency of oscillations $\omega \approx 2 \cdot 10^{16}\ \text{sec}^{-1}$.

a. Present an expression for the dipole moment d of the atom in this model.
b. Estimate the rate of the energy loss $\frac{dW}{dt}$ by the atom due to radiation of electromagnetic waves. Present an analytical expression and give also the numerical value (the electron charge is $e = 1.6 \cdot 10^{-19}$ Coulomb.)
c. Describe briefly, qualitatively the angular distribution of the radiated waves. Present then the formula that describes this distribution.

d. Estimate the force, which the radiated electromagnetic waves produce on an atom during the dipole radiation. Remember that the force equals the variation of the momentum per second. Therefore, if you know the rate of the radiated momentum (i.e. momentum radiated per second) you also know the force.
(Hint: this is a simple trap, to avoid it remember part Q4 c.)

e. Compare this energy loss with a typical atomic energy (which is $13.6 \text{ eV} \simeq 2.2 \cdot 10^{-18} J$). Derive from this comparison an estimate for an interval of time τ during which the atomic electron radiates all its energy (and falls down on the nucleus). Present this interval via an analytic expression and also give the numerical value.

Hint.
If you do not remember the formula for the radiation power of a dipole, then you can derive it from simple dimensional analyses. Remember that the rate of the radiated energy depends only on the dipole moment d and the frequency ω (as well as on constants $c, 4\pi\varepsilon_0$). A numerical factor in this formula cannot be extracted this way, but this factor is not important, when an estimate is considered.