THE UNIVERSITY OF NEW SOUTH WALES

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SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2009

PHYS3210

QUANTUM MECHANICS

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions ARE of equal value

Candidates must provide their own,
university approved, calculator.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Candidates may keep this paper.

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Examination 2009
Quantum Mechanics PHYS3210
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$$\hbar = 1.054 \times 10^{-34} Js$$

$$1eV = 1.602 \times 10^{-19} J \quad .$$

Question 1

Consider a particle in an energy split double well potential shown in Fig.1. You might recall a

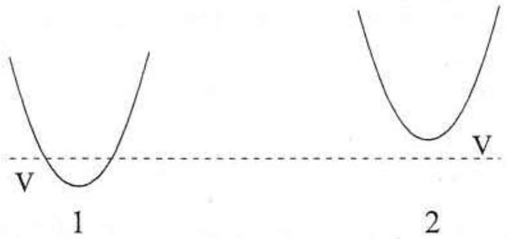

FIG. 1:

question from your midsession test. The effective Hamiltonian describing the system is

$$H_{eff} = \begin{pmatrix} -v & w \\ w & v \end{pmatrix} , \tag{1}$$

where $-v$ is the energy shift of the first well, $+v$ is the energy shift of the second well, and w is the particle tunneling amplitude between the wells. Assume that $v \gg w$, this assumption is not important physically, but it simplifies the algebra.

a) (5marks) Find eigenenergies and eigenstates of the system.

At $t = 0$ the system is prepared in the quantum state with wave function

$$\psi(t = 0) = \psi_1 \ .$$

So initially the probability to locate particle in the first well is 1 and the probability to locate particle in the second well is 0.

b) (5marks) Find a probability $P_2(t)$ to locate particle in the second well at time t.

Question 2

a) The spherical harmonics $|l,m\rangle = Y_{lm}$ are eigenfunctions of the operators $\hat{l}^2 = \hat{L}^2/\hbar^2$ and $\hat{l}_z = \hat{L}_z/\hbar$.

(i) **(1mark)** Write down eigenvalue equations for the two operators, showing clearly the possible values of l and m.

(ii) **(1mark)** Because of rotational symmetry

$$\langle \hat{l}_x^2 \rangle = \langle \hat{l}_y^2 \rangle \ ,$$

where $\langle ... \rangle = \langle lm | ... | lm \rangle$. Using the explicit form of $\hat{l}^2$, $\hat{l}^2 = \hat{l}_x^2 + \hat{l}_y^2 + \hat{l}_z^2$, prove that

$$\langle \hat{l}_x^2 \rangle = \frac{1}{2}[l(l+1) - m^2] \ .$$

(iii) **(2marks)** Using the raising and lowering operators $\hat{l}_\pm = \hat{l}_x \pm i\hat{l}_y$ show that

$$\langle \hat{l}_x \rangle = \langle \hat{l}_y \rangle = 0 \ .$$

(Remember that $\hat{l}_\pm$ acting on $|lm\rangle$ always changes value of m.)

b) The first three rotational energy levels of the PbO molecule are

$$\begin{aligned} \epsilon_0 &= 0, \\ \epsilon_1 &= 0.76 \times 10^{-4} eV, \\ \epsilon_2 &= 2.28 \times 10^{-4} eV. \end{aligned}$$

The molecule consists of isotopes ^{208}Pb and ^{16}O, $M_O = 26.5 \times 10^{-27} kg$, $M_{Pb} = 343 \times 10^{-27} kg$.

(i) **(2marks)** What is the degeneracy of each level? Write wave functions of all quantum states corresponding to these levels in terms of spherical harmonics Y_{lm}.

(ii) **(4marks)** Using the energy levels find the moment of inertia I of the molecule and the distance between the Lead and the Oxygen nuclei.

Question 3

The ground state energy of hydrogen atom is

$$E = -\frac{1}{2}\frac{me^4}{(4\pi\epsilon_0\hbar)^2} \approx -13.6eV\ .$$

Here m is the electron mass and e is the elementary charge. The electron wave function of the state reads

$$\psi(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B},$$

where

$$a_B = \frac{4\pi\epsilon_0\hbar^2}{me^2} \approx 0.529\ 10^{-10}m,$$

is the Bohr radius.

Consider a He* atom that consists of the He nucleus (two protons and two neutrons), a muon, and an electron. Muon is an elementary particle that has an electric charge equal to that of electron, but muon is 207 times heavier than electron, $m_\mu = 207m$.

a) (3 marks) Calculate energy required to remove electron from the atom.

b) (3 marks) Calculate energy required to remove muon from the atom.

c) (2 marks) Find the ground state wave function of electron.

d) (2 marks) Find the ground state wave function of muon.

Question 4

The spin operator $\hat{\vec{s}}$ for a particle with spin 1/2 (for example an electron or a neutron) can be expressed in terms of Pauli matrixes $\hat{\vec{\sigma}}$, $\hat{\vec{s}} = \frac{1}{2}\hat{\vec{\sigma}}$,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

The wave function of a particle polarized along the z-axis is determined by the equation

$$\hat{s}_z\psi_1 = +\frac{1}{2}\psi_1 ,$$

the wave function of a particle polarized along the x-axis is determined by the equation

$$\hat{s}_x\psi_2 = +\frac{1}{2}\psi_2 ,$$

and the wave function of a particle polarized along the y-axis is determined by the equation

$$\hat{s}_y\psi_3 = +\frac{1}{2}\psi_3 .$$

a) (5marks) Find the wave functions ψ_1, ψ_2, and ψ_3. Do not forget to normalize the functions.

A neutron beam at the Lucas Heights nuclear reactor has flux 1.6 10^{10}*neutrons/sec*, and all the neutrons are polarized along the z-axis which is directed vertically up. The beam is transmitted through two Stern-Gerlach polarizers. The first polarizer transmits only neutrons polarized along the x-axis that is directed from South to North. The second polarizer transmits only neutrons polarized along the y-axis that is directed from East to West.

b) (5marks) Find intensity of the transmitted beam. Explain your answer. *Hint: results of the part* **a** *might be helpful.*