THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2008

PHYS3210

QUANTUM MECHANICS

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions ARE of equal value

Candidates may not bring their own calculators.

The following materials will be provided by the Enrolment and Assessment Section: Calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Candidates may keep this paper.

PPS

Examination 2008
Quantum Mechanics PHYS3210
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The questions are of equal value.
Calculators may be used.
This paper may be retained by the candidate.

$$\hbar = 1.054 \times 10^{-34} Js$$

$$1eV = 1.602 \times 10^{-19} J \quad .$$

Question 1

a) (5marks) A particle of mass m moves in a one-dimensional potential $V(x) = g\delta(x)$, where $g = -|g|$ is negative. Find the wave function and the energy of the bound state, do not forget to normalize the wave function. You can use without proof the matching conditions at the δ-function potential.

$$\psi_+ = \psi_- \ , \quad \psi'_+ - \psi'_- = \frac{2mg}{\hbar^2}\psi \ .$$

b) (5marks) Consider a diamond at zero temperature. In some approximation we can model the crystal saying that every Carbon atom moves in the attractive δ-function potential with $|g| \approx 0.06eV \times 10^{-11}meter \approx 10^{-31}Joule \times meter$. This is the 1D model of the crystal. The mass of the Carbon atom is $m \approx 2 \times 10^{-26}kg$. Calculate the root mean square quantum fluctuation of the Carbon atom position, $r_{rms} = \sqrt{\langle\psi|x^2|\psi\rangle}$. Derive a formula for r_{rms}, and find value of the rms fluctuation in meters.

Question 2

a) (3marks) The operator of angular momentum is defined as

$$\hat{\vec{L}} = \hat{\vec{r}} \times \hat{\vec{p}} \ , \qquad \hat{\vec{l}} = \hat{\vec{L}}/\hbar \ .$$

Calculate commutators

$$[\hat{l}_x, \hat{l}_y] \ , \quad [\hat{l}_y, \hat{l}_z] \ , \quad [\hat{l}_z, \hat{l}_x] \ .$$

b) (2marks) The spherical harmonics $|l, m\rangle = Y_{lm}$ are eigenfunctions of the operators $\hat{l}^2 = \hat{L}^2/\hbar^2$ and $\hat{l}_z = \hat{L}_z/\hbar$.

Write down eigenvalue equations for the two operators, showing clearly the possible values of l and m.

c) The first three rotational energy levels of the PbO molecule are

$$\begin{aligned} \epsilon_0 &= 0, \\ \epsilon_1 &= 0.76 \times 10^{-4} eV, \\ \epsilon_2 &= 2.28 \times 10^{-4} eV. \end{aligned}$$

The molecule consists of isotopes ^{208}Pb and ^{16}O, $M_O = 26.5 \times 10^{-27}kg$, $M_{Pb} = 343 \times 10^{-27}kg$.

(i) (2marks) What is the degeneracy of each level? Write wave functions of all quantum states corresponding to these levels in terms of spherical harmonics Y_{lm}.

(ii) (3marks) Using the energy levels find the moment of inertia I of the molecule and the distance between the Lead and the Oxygen nuclei.

Question 3

The ground state energy of the hydrogen atom is

$$E = -\frac{1}{2}\frac{me^4}{(4\pi\epsilon_0\hbar)^2} \approx -13.6eV \ .$$

Here m is the electron mass and e is the elementary charge. The electron wave function of the state reads

$$\psi(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B} ,$$

where $a_B = \frac{4\pi\epsilon_0\hbar^2}{me^2} \approx 0.529\ 10^{-10}m$ is the Bohr radius.

Consider a hydrogen-like Oxygen ion, i. e. an ion that consists of the Oxygen nucleus and single electron. The charge of the nucleus is $Q = Ze$, where $Z = 8$. Based on the Schroedinger equation for an electron in the Coulomb field show how the ground state energy and the ground state wave function scale with Z. Hence calculate

a) (3 marks) The ground state energy of the oxygen ion, present both the expression in terms of fundamental constants and the numerical value of the energy in eV.

b) (3 marks) The electron wave function in the oxygen ion ground state.

Consider a muonium atom. The muonium is similar to the hydrogen, but the electron is replaced by a muon. So the muonium atom consists of proton and muon. The muon is an elementary particle that has an electric charge equal to that of the electron, but muon is 207 times heavier than electron, $m_\mu = 207m$.

c) (4 marks) Find the binding energy of the muonium atom and estimate its size.

Question 4

The spin operator $\hat{\vec{s}}$ for a particle with spin 1/2 (for example an electron) can be expressed in terms of Pauli matrixes $\hat{\vec{\sigma}}$, $\hat{\vec{s}} = \frac{1}{2}\hat{\vec{\sigma}}$,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

The wave function of a particle polarized along the z-axis is determined by the equation

$$\hat{s}_z\psi_1 = +\frac{1}{2}\psi_1 ,$$

and the wave function of a particle polarized along the x-axis is determined by the equation

$$\hat{s}_x\psi_2 = +\frac{1}{2}\psi_2 ,$$

a) (5marks) Find the wave functions ψ_1 and ψ_2. Do not forget to normalize the functions.

The neutron beam at the Lucas Heights nuclear reactor has flux $1.6\ 10^{10} neutrons/sec$, and all the neutrons are polarized along z-axis which is directed vertically up. The beam is transmitted through the Stern-Gerlach polarizer that transmits only neutrons polarized along x-axis that is directed from South to North. Spin of the neutron is 1/2.

b) (5marks) Find intensity of the transmitted beam. Explain your answer. *Hint: results of the part* **a** *might be helpful.*