THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2012

PHYS3080 Solid State Physics

PHYS3021 Statistical and Solid State Physics – Paper 2

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions are NOT of equal value

This paper may be retained by the candidate.

Students must provide their own UNSW approved calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Data and Formula Sheet

N.B. This is a generic PHYS3080/PHYS3021 data/formula sheet

$\mathbf{a}^* = \frac{2\pi(\mathbf{b}\mathrm{x}\mathbf{c})}{\mathbf{a}.(\mathbf{b}\mathrm{x}\mathbf{c})}$ and cyclic permutation of numerator

$e^x = 1 + x + \frac{x^2}{2}...$ $\int_0^{\Theta_D/T}\left(\frac{x^4 e^x dx}{(e^x - 1)^2}\right) \cong \int_0^{\infty}\left(\frac{x^4 e^x dx}{(e^x - 1)^2}\right) = \frac{4\pi^4}{15}$

$\dot{Q} = \frac{dQ}{dt} = \kappa A \frac{dT}{dx}$ $C_v = 1/2\, k_B T\, mol^{-1}$ per degree of freedom

$\kappa = \frac{1}{3}\bar{v} l C$ $R = k_B/N_A$ $E_{th} = k_B T$

$\varepsilon = E_g + \frac{\hbar^2 k^2}{2m_e}$ $\varepsilon = -\frac{\hbar^2 k^2}{2m_h}$ $E_n = -\frac{m_e^* e^4}{8h^2 n^2 \varepsilon_0^2}$ $a = a_0 \varepsilon_r \left(\frac{m_e}{m_e^*}\right)$ $a_0 = 0.053\,\mathrm{nm}$

$n_n p_n = n_i^2 = n_p p_p$ $R_H = -\frac{1}{ne}$ $n_i = p_i = (N_c N_v)^{1/2} \exp(-E_g/2k_B T)$

$np = (N_c N_v)\exp(-E_g/k_B T)$

$n \approx N_c \exp(-E_D/k_B T)$ for $k_B T << E_D$ $p \approx N_v \exp(-E_A/k_B T)$ for $k_B T << E_A$

$\mathbf{F} = q(\mathbf{v}\mathrm{x}\mathbf{B})$ $I = nAve$ $\mathbf{v} = -\frac{e\tau}{m_e}\mathbf{E}$ $\mathbf{J} = \sigma\mathbf{E}$ $\sigma = ne\mu = \frac{ne^2\tau}{m}$

$\mu = \frac{v_d}{E}$

$e = 1.6\mathrm{x}10^{-19}\,C$ $\varepsilon_0 = 8.854\mathrm{x}10^{-12}\,Fm^{-1}$ $N_A = 6.023\mathrm{x}10^{26}\,(kg.mol)^{-1}$

$h = 6.63\mathrm{x}10^{-34}\,Js$ $\hbar = 1.05\mathrm{x}10^{-34}\,Js$ $\hbar^2 = 1.11\mathrm{x}10^{-68}\,J^2s^2$ $\lambda_{visible} \sim 400 - 700\mathrm{nm}$

$v = \frac{1}{\hbar}\frac{d\varepsilon}{dk_x}$ $m^* = \hbar^2 \Big/ \frac{d^2\varepsilon}{dk_x^2}$ $j = j_0 \sin\left[\frac{2e}{\hbar}\left(V_0 t + \frac{v}{\omega}\sin(\omega t)\right) + \delta_0\right]$

$V_0 = \frac{n\hbar\omega}{2e} = \frac{nhv}{2e}$

$n_{phonon} \sim \exp(-\Theta_D/T)$ $\lambda_{phonon} \sim \exp(+\Theta_D/T)$

$k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$ $\xi_0 = \frac{\hbar v_F}{\pi\Delta(0)}$ $V_0 = \frac{n\hbar\omega}{2e} = n\nu\Phi$

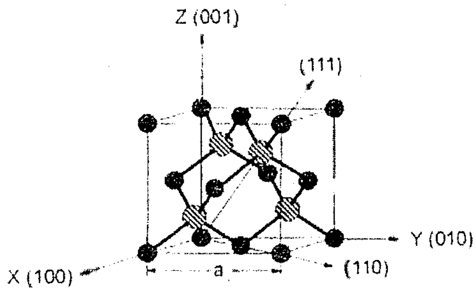
Question 1 (31 Marks)

(a) Consider the crystal structure shown in the figure below, right.

(i) What is the name given to this structure? (1 mark)

(ii) What is the Bravais lattice type? (1 mark)

(iii) If a set of primitive lattice vectors for this Bravais lattice type is



$$\mathbf{a}_1 = \frac{a}{2}(\hat{y}+\hat{z}) \quad \mathbf{a}_2 = \frac{a}{2}(\hat{z}+\hat{x}) \quad \mathbf{a}_3 = \frac{a}{2}(\hat{x}+\hat{y}),$$

calculate the reciprocal lattice vectors; describe the resulting lattice and state how it is related to the real-space lattice. (8 marks)

(b) Given that the density of states function for a free electron gas is

$$g(\varepsilon) = \frac{V}{2\pi^2\hbar^3}(2m)^{3/2}\varepsilon^{1/2}$$

derive an expression for the Fermi energy, ε_F, and the Fermi wave vector, k_F, in terms of the number of electrons per unit volume. (8 marks)

(c) The total number of occupied electron states, $N(\varepsilon)$, in the energy range $\varepsilon \to \varepsilon + d\varepsilon$ is given by the product of the occupation probability $f(\varepsilon)$ with the density states function $g(\varepsilon)$. Sketch the form of the three quantities $N(\varepsilon)$, $f(\varepsilon)$, $g(\varepsilon)$ for a simple free electron metal. Indicate the situation for $T = 0K$ and $T_F >> T >> 0K$, where T_F is the Fermi temperature. (Put both curves on one plot or use a separate plot for each, as you prefer.) (6 marks)

(d) Give a concise explanation of the reason the observed electronic (i.e. the conduction electrons) contribution to the heat capacity of a metal is only a small fraction of that expected classically. Include a sketch illustrating your answer. (4 marks)

(e) Using your result from part (b), calculate $\varepsilon_{F,0}$ for aluminium metal (Al is *trivalent* with density $\rho_{Al} = 2.70\text{x}10^3\ \text{kgm}^{-3}$ and atomic mass $26.98\ \text{kg(kmole)}^{-1}$); give your answer in electron volts. (3 marks)

Question 2 (26 Marks)
Consider a crystal for which the $\varepsilon - k$ relation is $\varepsilon = \varepsilon_1 + (\varepsilon_2 - \varepsilon_1)\sin^2(ak_x/2)$, with $\varepsilon_2, \varepsilon_1$ constants.

(a) Sketch and label the $\varepsilon - k$ dispersion relation for the first two Brillouin zones. (5 marks)

(b) Populate the band with one electron (i.e. assume one electron in the band) and, ignoring scattering of the electron, sketch and describe the behaviour of

(i) the effective mass, (6 marks) and,

(ii) the electron velocity, (6 marks),

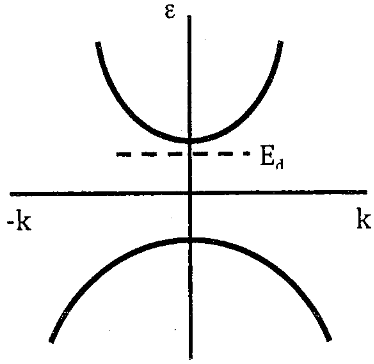
in a constant applied electric field E. Sketch each on a separate graph. Your sketch should include the range $-\pi/a < k_x \leq \pi/a$.

(c) Briefly describe the motion of the electron in this band when a constant electric field E is applied. (6 marks)

(d) Comment on the change to (c) if scattering is taken into account. (3 marks)

Question 3 (21 Marks)

(a) The diagram at right shows schematically the energy bands of an n-type doped semiconductor. This semiconductor has electron effective mass $0.067m_e$ and dielectric constant 13.1. Assuming the bound donor electrons to be in Bohr orbits (hydrogenic impurity model),



(i) calculate the first donor ionization energy within the Bohr picture. (4 marks) and,

(ii) given that the semiconductor exhibits a sharp increase in electrical conductivity when illuminated with radiation of wavelength $\lambda \leq 860$ nm find E_d referenced to the top of the valence band. (3 marks)

(b) The valence band shown in the figure above may be taken to be parabolic and is described by $\varepsilon(k) = -10^{-37}k^2$ J near to the band edge. An empty state at $\mathbf{k} = 10^9\hat{\mathbf{k}}_x$ m^{-1} provides a mobile hole. For the mobile hole calculate, paying particular attention to the sign,

(i) the effective mass, (4 marks)

(ii) the energy, (4 marks)

(iii) the momentum, (3 marks)

(iv) the velocity. (3 marks)

Question 4 (20 Marks)
Write brief notes (about 2-3 pages for each, including diagrams and any equations you include, but no more than this) on ***two only*** from the list of four topics given below.

Use simple diagrams and/or sketch graphs to illustrate your answers, where appropriate, ensuring that you label these and refer to them in your account.

Choose ***two only*** from the following four (a) - (d)

(a) Thermal conduction in crystalline solids (10 marks)
(b) Heat capacity of solids. Your answer must discuss (but is not limited to) 'classical' theory (law of Dulong and Petit), the Debye and Einstein phonon models and the electronic (electron) contribution to the heat capacity. (10 marks)
(c) Effect of dopant concentration and temperature on the electrical conductivity of semiconductor materials. The relevant sketch graphs must be included in your answer. (10 marks)
(d) The BCS theory of superconductivity. You may use information from the data sheet in your answer, as required. (10 marks)