THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE, 2010

PHYS3080

Solid State Physics

Time Allowed – 2 hours

Total number of questions - 5

Answer ALL questions

All questions are NOT of equal value

This paper may be retained by the candidate.

Students must provide their own UNSW approved calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Data and Formula Sheet

N.B. This is a generic PHYS3080 data/formula sheet and contains some additional information you may not necessarily need for this exam

$\mathbf{a}^* = \dfrac{2\pi(\mathbf{bxc})}{\mathbf{a}.(\mathbf{bxc})}$ and cyclic permutation of numerator

$$e^x = 1 + x + \frac{x^2}{2}\ldots \qquad \int_0^{\Theta_D/T}\left(\frac{x^4 e^x dx}{(e^x-1)^2}\right) \cong \int_0^{\infty}\left(\frac{x^4 e^x dx}{(e^x-1)^2}\right) = \frac{4\pi^4}{15}$$

$$\dot{Q} = \frac{dQ}{dt} = \kappa A\frac{dT}{dx} \qquad C_v = 1/2\,k_B T\ \mathrm{mol}^{-1} \text{ per degree of freedom}$$

$$\kappa = \frac{1}{3}\bar{v}lC \qquad R = k_B/N_A \qquad E_{th} = k_B T$$

$$\varepsilon = E_g + \frac{\hbar^2 k^2}{2m_e} \qquad \varepsilon = -\frac{\hbar^2 k^2}{2m_h} \qquad E_n = -\frac{m_e^* e^4}{8h^2 n^2 \varepsilon_0{}^2} \qquad a = a_0\varepsilon_r\left(\frac{m_e}{m_e^*}\right) \quad a_0 = 0.053\ \mathrm{nm}$$

$$n_n p_n = n_i{}^2 = n_p p_p \qquad R_H = -\frac{1}{ne} \qquad n_i = p_i = (N_c N_v)^{1/2}\exp(-E_g/2k_B T)$$

$$np = (N_c N_v)\exp(-E_g/k_B T)$$

$$n \approx N_c \exp(-E_D/k_B T) \text{ for } k_B T << E_D \qquad p \approx N_v \exp(-E_A/k_B T) \text{ for } k_B T << E_A$$

$$\mathbf{F} = q(\mathbf{v}\mathrm{x}\mathbf{B}) \qquad I = nAve \qquad \mathbf{v} = -\frac{e\tau}{m_e}\mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E} \qquad \sigma = ne\mu = \frac{ne^2\tau}{m}$$

$$\mu = \frac{v_d}{E}$$

$$e = 1.6\mathrm{x}10^{-19}\ \mathrm{C} \qquad \varepsilon_0 = 8.854\mathrm{x}10^{-12}\ \mathrm{Fm}^{-1} \qquad N_A = 6.023\mathrm{x}10^{26}\ (\mathrm{kg.mol})^{-1}$$

$$h = 6.63\mathrm{x}10^{-34}\ \mathrm{Js} \qquad \hbar = 1.05\mathrm{x}10^{-34}\ \mathrm{Js} \qquad \hbar^2 = 1.11\mathrm{x}10^{-68}\ \mathrm{J^2s^2} \qquad \lambda_{visible} \sim 400-700\mathrm{nm}$$

$$v = \frac{1}{\hbar}\frac{d\varepsilon}{dk_x} \qquad m^* = \hbar^2\Big/\frac{d^2\varepsilon}{dk_x^2} \qquad j = j_0\sin\left[\frac{2e}{\hbar}\left(V_0 t + \frac{v}{\omega}\sin(\omega t)\right) + \delta_0\right]$$

$$V_0 = \frac{n\hbar\omega}{2e} = \frac{nh\nu}{2e}$$

$$n_{phonon} \sim \exp(-\Theta_D/T) \qquad \lambda_{phonon} \sim \exp(+\Theta_D/T)$$

$$k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3} \qquad \xi_0 = \frac{\hbar v_F}{\pi\Delta(0)} \qquad V_0 = \frac{n\hbar\omega}{2e} = n\nu\Phi$$

Question 1 (20 Marks)

In a solid the mean energy per oscillator (or phonon energy) is given by

$$\bar{\varepsilon} = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\left(e^{\hbar\omega/k_BT} - 1\right)}$$

(i) What does the first term on the right, $\frac{1}{2}\hbar\omega$, represent? **(2 marks)**

(ii) Explain briefly what the factors $\hbar\omega$ and $\frac{1}{\left(e^{\hbar\omega/k_BT} - 1\right)}$ in the second term represent. **(4 marks)**.

(iii) Use this expression to show that, in thermal equilibrium at temperature T, the energy of a sufficiently long wavelength mode is k_BT. **(5 marks)**

(iv) Estimate the number of modes that will be excited at temperatures $T << \theta_D$. **(4 marks)**

(v) Thus show that for $T << \theta_D$ the heat capacity is given approximately by $C_V \sim Nk_B(T/\theta_D)^3$ **(5 marks)**.

Question 2 (20 Marks)

(a) The Fermi-Dirac distribution function

$$f(\varepsilon)=\frac{1}{1+\exp\left(\frac{\varepsilon-\varepsilon_F}{k_B T}\right)}$$

gives the state occupation probability for electrons in a free electron metal.

(i) Define all symbols in this expression. **(3 marks)**

(ii) The total number of occupied electron states, $N(\varepsilon)$, in the energy range $\varepsilon \rightarrow \varepsilon + d\varepsilon$ is given by the product of the occupation probability $f(\varepsilon)$ with the density states function $g(\varepsilon)$. Sketch the form of the three quantities $N(\varepsilon)$, $f(\varepsilon)$, $g(\varepsilon)$ for a simple free electron metal. Indicate the situation for $T = 0K$ and $T_F >> T >> 0K$, where T_F is the Fermi temperature. (Put both curves on one plot or use a separate plot for each, as you prefer.) **(6 marks)**

(b) Give a concise explanation of the reason the observed electronic (i.e. the conduction electrons) contribution to the heat capacity of a metal is only a small fraction of that expected classically. Include a sketch illustrating your answer. **(4 marks)**

(c) The Fermi energy at 0K is given by $\varepsilon_{F,0} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$.

(i) Calculate $\varepsilon_{F,0}$ for aluminium metal (Al is *trivalent* with density $\rho_{Al} = 2.70\text{x}10^3\ \text{kgm}^{-3}$ and atomic mass $26.98\ \text{kg}(\text{kmole})^{-1}$); give your answer in electron volts. **(3 marks)**

(ii) Determine the Fermi velocity and the de Broglie wavelength of an electron moving in aluminium at the Fermi energy. **(2 marks)**

(d) A particular sample of aluminium has drift velocity $v_d = 2.16\ \text{ms}^{-1}$ in an electric field $E = 500\ \text{Vm}^{-1}$. Estimate (i) the electron mobility; (ii) the relaxation (scattering) time. **(2 marks)**

Question 3 (25 Marks)

(a) For electrons moving in the energy band of a one-dimensional chain of atoms we have the two important results

(1) $v = \dfrac{d\omega}{dk} = \dfrac{1}{\hbar}\dfrac{d\varepsilon}{dk}$ velocity of an electron wavepacket

and,

(2) $\delta\varepsilon = -eE\delta x$ wavepacket in a uniform, static electric field E.

(i) Use results (1) and (2) to show that $\hbar\dfrac{dk}{dt} = -eE$; show your working.

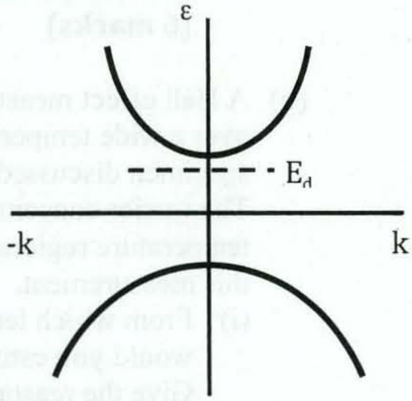
(3 marks)

(ii) What is the quantity $\hbar k$? Explain briefly, 3-4 lines only. **(2 marks)**

(iii) Take the time derivative of (1) together with the result of part (i) to show that $m_e = \hbar^2 \Big/ \dfrac{d^2\varepsilon}{dk^2}$. **(3 marks)**

(iv) What is the quantity m_e? Explain briefly, 3-4 lines only. **(3 marks)**

(b) The diagram at right shows schematically the energy bands of an n-type doped semiconductor. This semiconductor has electron effective mass $0.067m_e$ and dielectric constant 13.1. Assuming the bound donor electrons to be in Bohr orbits (hydrogenic impurity model),



(i) calculate the first donor ionization energy within the Bohr picture. **(3 marks)**

(ii)) Given that the semiconductor exhibits a sharp increase in electrical conductivity when illuminated with radiation of wavelength $\lambda \leq 860$ nm find E_d referenced to the top of the valence band. **(3 marks)**

(c) The valence band shown in the figure above may be taken to be parabolic and is described by $\varepsilon = -10^{-37}k^2$ J near to the band edge. An empty state at $\mathbf{k} = 10^9\hat{\mathbf{k}}_x$ m^{-1} provides a mobile hole. For the mobile hole calculate, paying particular attention to the sign,

(i) the effective mass **(2 marks)**

(ii) the energy **(2marks)**

(iii) the momentum **(2 marks)**

(iv) the velocity **(2 marks)**

Question 4 (15 Marks)

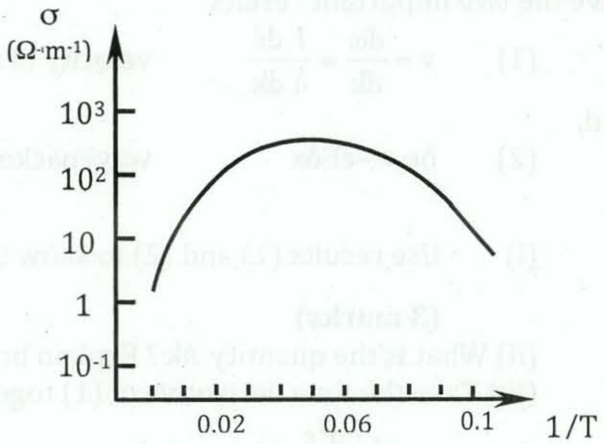
(a) The electrical conductivity σ of a doped semiconductor specimen is measured over a wide temperature range with the following features being observed:

(i) At low temperatures σ rises with increasing temperature.

(ii) At intermediate temperatures σ falls with increasing temperature.

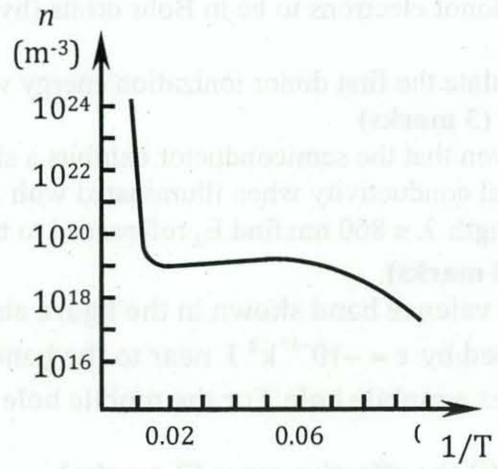
(iii) At high temperatures σ increases rapidly with increasing temperature.

The behaviour is illustrated on the graph at right; region (iii) is not shown.



Give the probable reasons for the behaviour in each temperature region. **(6 marks)**

(b) A Hall effect measurement is performed over a wide temperature range on the specimen discussed in part (a) above. The carrier concentration *n* in different temperature regions is determined from this measurement.



(i) From which temperature region would you estimate the band gap E_g? Give the reasons for your choice and state what you would plot graphically to find a value for E_g. **(5 marks)**

(ii) From which temperature region would you estimate the net donor concentration N_D-N_A? Give the reasons for your choice and estimate the value of N_D-N_A for this specimen. (**4 marks**)

Question 5 (20 Marks)

Write brief notes (about 2-3 pages for each, including diagrams and any equations you include, but no more than this) on ***two only*** from the list of four topics given below.

Use simple diagrams and/or sketch graphs to illustrate your answers where appropriate ensuring that you label these and refer to them in your account.

(a) Crystalline solids. Your answer must discuss (but is not limited to) lattice and basis, bravais lattices and symmetry, unit cells, packing fraction. **(10 marks)**
(b) The BCS theory of superconductivity. Your account must include a description of the isotope effect, electron pairing and energy considerations, virtual phonon exchange and the superconducting energy gap. **(10 marks)**
(c) Effect of dopant concentration and temperature on the electrical conductivity of semiconductor materials. The relevant sketch graphs must be included in your answer. **(10 marks)**
(d) The Josephson effects. You may use information from the data sheet and refer to the diagram included below. **(10 marks)**