THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS3080 SOLID STATE PHYSICS

FINAL EXAMINATION
SESSION 1 – JUNE 2009

1. Time allowed – 2 hours

2. Reading time – 10 minutes

3. This examination paper has 4 pages.

4. Total number of questions – 5

5. Total number of marks – 60

6. All questions are NOT of equal value. Marks available for each question are shown in the examination paper.

7. Answer all questions.

8. Students are required to supply their own University approved calculator.

9. Students may bring one A4-sized page of notes into the exam. Notes must be handwritten only and both sides of the page may be used.

10. All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

11. This paper may be retained by the candidate.

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The following information is supplied as an aid to memory.

Planck's constant $h = 6.626 \times 10^{-34}$ Js or $\hbar = 1.054 \times 10^{-34}$ Js

Fundamental charge unit $e = 1.60 \times 10^{-19}$ C   Speed of light (vacuum) $c = 3.0 \times 10^{8}$ m/s

Electron mass $m_e = 9.1 \times 10^{-31}$ kg   Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ JK^{-1}

Permittivity constant $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1} Permeability constant $\mu_0 = 4\pi \times 10^{-7}$ VsA^{-1}m^{-1}

Avogadro's number $N_A = 6.023 \times 10^{23}$ mol^{-1} Ideal gas constant $R = k_B/N_A = 8.314$ Jmol^{-1}K^{-1}

Bohr Magneton $\mu_B = 9.274 \times 10^{-24}$ JT^{-1}

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}} \qquad \int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15} \qquad \int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2} dx = \frac{4\pi^4}{15}$$

$$\mathbf{a}^* = \frac{2\pi(\mathbf{b} \times \mathbf{c})}{|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|} \qquad e^x = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n!}$$

$\sin(2\theta) = 2\sin\theta\cos\theta$ Bragg's law: $n\lambda = 2d\sin\theta$

$m^*_{Si} = 0.43\, m_e$ $E_g^{Si} = 1.1$ eV $E_D^{P} = 45$ meV

3D Density of States: $g(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$

Pauli paramagnetic susceptibility: $\chi = \frac{3N\mu_B^{\,2}\mu_0}{2E_F}$

Hydrogen atom Bohr model results: $E_n = \frac{me^4}{8\varepsilon_0 h^2}\frac{1}{n^2}$ $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$

Intrinsic carrier density: $n = N_C^{eff} e^{-E_g/2kT}$ where $N_C^{eff} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$

Doped semiconductor carrier density: $n = \sqrt{N_C N_D}\, e^{-E_D/2kT}$

Hall coefficient: $R_H = -(ne)^{-1}$

Debye lattice energy: $U = \int_0^{\omega_D} \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} g(\omega) d\omega$

Question 1 [13 Marks]

(a) Draw sketches illustrating the crystal structures of the simple cubic, face centered cubic and body centered cubic lattices. You may use conventional cells for your sketches, whatever is easiest to visualise. Highlight with captions any key features that may not be obvious in your sketches.

(b) You are given two pieces of transparent material which to the eye appear identical in all respects. One is a crystal and the other is a glass. Briefly describe an experiment for distinguishing between the crystal and the glass. For bonus marks, describe a second experiment that could be used as further confirmation.

(c) Consider a crystal in which the atoms are located at points defined by $\boldsymbol{R} = a(n\boldsymbol{i} + l\boldsymbol{j} + m\boldsymbol{k})$ where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are three orthonormal vectors, a is the lattice parameter, and n, l, m are integers.

(i) Name the lattice structure for this crystal.

(ii) Briefly define the Wigner-Seitz cell. What is its significance? Describe the shape of the Wigner-Seitz cell for this crystal (you may use sketches if this is helpful).

(iii) Write down the Miller indices for all of the planes that form boundaries for the Wigner-Seitz cell that surrounds the atom located at the origin $\boldsymbol{R} = 0$.

(iv) What is the crystal structure of the reciprocal lattice?

(v) Describe the shape and location of the first Brillouin zone for this crystal.

Question 2 [10 Marks]

Consider a ***two-dimensional*** square lattice with lattice parameter a, dimensions $l \times l$, and with one atom of mass m per lattice point interacting only with nearest neighbours with force constant γ. You may take the phonon dispersion curve to be $\omega = 2(\gamma/m)|\sin \frac{1}{2}ka|$.

(a) In the long wavelength limit, obtain the density of phonon states $g(\omega) = dN/d\omega$ (i.e., the number of lattice vibration modes per frequency interval $d\omega$).

(b) At high temperature ($k_BT >> \hbar\omega$), find the mean square displacement of an atom from its equilibrium position.

(c) Use your answer to (b) to comment on the stability of a two-dimensional crystal as the temperature is increased.

Question 3 [9 Marks]

Metallic sodium crystallises in body-centered cubic form, the length of the cube being 4.25×10^{-8} cm.

(a) Find the concentration of conduction electrons n assuming one conduction electron per atom.

(b) Adopting the free electron Fermi gas model for the conduction electrons, obtain an expression for the Fermi energy at T = 0K, and show that it depends only on the concentration of conduction electrons, and not on the mass of the crystal.

(c) Calculate the corresponding Fermi energy E_F.

(d) By calculating the density of states at the Fermi level, estimate the percentage of the total electrons that are able to contribute to the electronic heat capacity of the sample of metallic sodium.

Question 4 [15 Marks]

(a) Consider a piece of n-type-doped semiconductor. Assume that the donor ionisation energy is 6.6×10^{-4} eV and the dielectric constant of the semiconductor is 17.

(i) Give a brief account for the Bohr model for a dopant in a semiconductor, focussing on why a dopant atom in a semiconductor can be considered like a Hydrogen atom. What are the two factors that need to be accounted for in adapting the Hydrogen atom Bohr model for use with dopants in semiconductors.

(ii) Using the information above, calculate the effective mass m^*.

(iii) Calculate the radius of the donor electron orbit.

(iv) Provide a rough estimate the donor doping concentration N_D at which an impurity band will begin to form. This will occur when the adjacent donor orbits begin to overlap.

(b) Show by brief calculation (include all your working) and explain why:

(i) Diamond ($E_g = 5.5$ eV) is a good insulator at 300K.

(ii) Diamond is transparent in the optical region (λ = 400-700 nm) whereas Silicon is not.

(c) The Hall coefficient of a specimen of Si was found to be $R_H = -7.35 \times 10^{-5}$ m^3C^{-1} for temperatures $100K < T < 400K$.

(i) Calculate the intrinsic carrier density of Si at room temperature.

(ii) Using this information or otherwise, state whether this specimen is intrinsic or extrinsic at room temperature, and, if extrinsic, state the type of doping (is it n-type or p-type doped?).

(iii) The electrical conductivity of this Si specimen was found to be $200(\Omega m)^{-1}$. Calculate the density and mobility of the charge carriers.

Question 5 [13 Marks]

(a) Magnetism in condensed matter manifests itself in various forms for different materials and at different temperatures. Four types of magnetic behaviour are: Ferromagnetism, Diamagnetism, Antiferromagnetism and Paramagnetism.

For an ideal example of a material exhibiting each of the above magnetic behaviour, plot the magnetic suspectibility as a function of temperature. Pay attention to the relative scales in your diagrams so that the relative strengths of the magnetic phenomena are presented. Give one example of a real material that approximates each ideal form of magnetism.

(b) Free electrons can also exhibit paramagnetic behaviour. In a few sentences, and a sketched diagram if needed, briefly explain why this occurs.

(c) Give a numerical estimate for the magnetic susceptibility $\chi = M/B$, where M is the magnetization and B is the applied magnetic field, for a Fermi free electron gas with $N = 4.7 \times 10^{22}$ cm^{-3} electrons at T = 300K. If I raise the temperature to 350K, how much will the susceptibility change by?