THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE/JULY 2008

**PHYS3080**

**Solid State Physics**

Time Allowed – 2 hours

Total number of questions - 5

Answer ALL questions

All questions are NOT of equal value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the Enrolment and Assessment Section: Calculators

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

PPS

**Data and Formula Sheet**

$$\dot{Q} = \frac{dQ}{dt} = \kappa A \frac{dT}{dx}$$

$$\kappa = \frac{1}{3}\bar{v} l C$$

$$\varepsilon = E_g + \frac{\hbar^2 k^2}{2m_e} \qquad \varepsilon = -\frac{\hbar^2 k^2}{2m_h} \qquad E_n = -\frac{m_e^* e^4}{32\pi^2 n^2 \varepsilon_r^2 \varepsilon_0^2 \hbar^2} \qquad n_n p_n = n_i^2 = n_p p_p$$

$$n = N_c e^{(\varepsilon_F - \varepsilon_G)/k_B T} \qquad N_c = 2\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} \qquad p = N_v e^{-\varepsilon_F/k_B T} \qquad N_v = 2\left(\frac{2\pi m_h k_B T}{h^2}\right)^{3/2}$$

$$\mathbf{F} = q(\mathbf{v}\mathrm{x}\mathbf{B}) \qquad I = nAve \qquad \mathbf{v} = -\frac{e\tau}{m_e}\mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E} \qquad \sigma = ne\mu$$

$$\varepsilon_0 = 8.854\mathrm{x}10^{-12}\ \mathrm{Fm^{-1}}$$

$$N_A = 6.023\mathrm{x}10^{26}\ (\mathrm{kg.mol})^{-1} \qquad h = 6.63\mathrm{x}10^{-34}\ \mathrm{Js} \qquad \hbar = 1.05\mathrm{x}10^{-34}\ \mathrm{Js} \qquad \hbar^2 = 1.11\mathrm{x}10^{-68}\ \mathrm{J^2 s^2}$$

$$j = j_0 \sin\left[\frac{2e}{\hbar}\left(V_0 t + \frac{v}{\omega}\sin(\omega t)\right) + \delta_0\right], \quad V_0 = \frac{n\hbar\omega}{2e} = \frac{nh\nu}{2e}$$

$$n_{phonon} \sim \exp(-\Theta_D/T) \qquad \lambda_{phonon} \sim \exp(+\Theta_D/T)$$

$$e = 1.6\mathrm{x}10^{-19}\ \mathrm{C} \qquad k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3} \qquad \varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \qquad \Delta\varepsilon\Delta\tau \geq \hbar/2$$

$$V_0 = \frac{n\hbar\omega}{2e} = n\nu\Phi \qquad j = j_0 \sin\left(\frac{2eV}{\hbar}t + \delta_0\right)\ (\mathrm{ac}) \qquad j = j_0 \sin\delta,\ j_0 = \frac{e\hbar n_s}{m} K \exp(-Kd)\ (\mathrm{dc})$$

$$V = 2\Delta(T)/e \qquad \frac{h}{2e} = \Phi_0 = 2.07\mathrm{x}10^{-15}\ \mathrm{VHz^{-1}} \qquad \nu = \frac{\omega}{2\pi} = \frac{2eV}{h}$$

**Question 1** (22 Marks)
In a solid the mean energy per oscillator (or phonon energy) is given by

$$\bar{\varepsilon} = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\left(e^{\hbar\omega/k_BT} - 1\right)}$$

(i) Explain briefly what the factors $\hbar\omega$ and $\frac{1}{\left(e^{\hbar\omega/k_BT} - 1\right)}$ in the second term represent. (4 marks)

(ii) What type of quantum statistics do phonons obey and why? (2 marks)

(iii) Use the expression for $\bar{\varepsilon}$ to show that, in thermal equilibrium at temperature T, the energy of a sufficiently long wavelength mode is $k_BT$. (4 marks)

(iv) Estimate the number of modes that will be excited at temperatures $T << \theta_D$. (5 marks)

(v) Thus show that for $T << \theta_D$ the heat capacity is given approximately by $C_V \sim Nk_B(T/\theta_D)^3$. (5 marks)

(vi) Comment briefly (1-2 lines) on the agreement between the form of $C_V$ in (vi) and experimental data for $C_V$ at low temperatures. (2 marks)

**Question 2** (20 Marks)

(a) The Fermi-Dirac distribution function

$$f(\varepsilon)=\frac{1}{1+\exp\left(\frac{\varepsilon-\varepsilon_F}{k_BT}\right)}$$

gives the state occupation probability for electrons in a free electron metal.

(i) Define all symbols in this expression. (2 marks)

(ii) The total number of occupied electron states, $N(\varepsilon)$, in the energy range $\varepsilon \rightarrow \varepsilon + d\varepsilon$ is given by the product of the occupation probability $f(\varepsilon)$ with the density states function $g(\varepsilon)$. Sketch the form of the three quantities $N(\varepsilon)$, $f(\varepsilon)$, $g(\varepsilon)$ for a simple free electron metal. Indicate the situation for $T = 0K$ and $T_F >> T >> 0K$, where $T_F$ is the Fermi temperature. (Put both curves on one plot or use a separate plot for each, as you prefer.) (6 marks)

(b) Give a concise explanation of the reason the observed electronic (i.e. the conduction electrons) contribution to the heat capacity of a metal is only a small fraction of that expected classically. Include a sketch illustrating your answer. (4 marks)

(c) The Fermi energy at 0K is given by $\varepsilon_{F,0}=\frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$.

(i) Calculate $\varepsilon_{F,0}$ for aluminium metal (Al is *trivalent* with density $\rho_{Al} = 2.70\text{x}10^3\ \text{kgm}^{-3}$ and atomic mass $26.98\ \text{kg}(\text{kmole})^{-1}$); give your answer in electron volts. (4 marks)

(ii) Determine the Fermi velocity and the de Broglie wavelength of an electron moving in aluminium at the Fermi energy. (2 marks)

(d) A particular sample of aluminium has drift velocity $v_d = 2.16\ \text{ms}^{-1}$ in an electric field $E = 500\ \text{Vm}^{-1}$. Estimate (i) the electron mobility; (ii) the relaxation (scattering) time. (2 marks)

**Question 3** (18 Marks)

For electrons moving in the energy band of a one-dimensional chain of atoms we have the two important results

(1) $$v = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{d\varepsilon}{dk}$$ velocity of an electron wavepacket

and

(2) $$\delta\varepsilon = -eE\delta x$$ motion of electron wavepacket in a uniform, static electric field E.

(i) Use results (1) and (2) to show that $\hbar\frac{dk}{dt} = -eE$; show all your working (4 marks)

(ii) What is the quantity $\hbar k$? Explain concisely, 3-4 lines only. (2 marks)

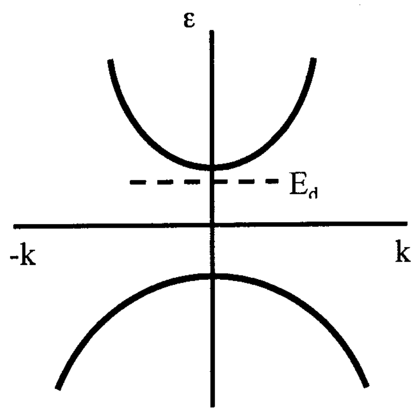
(iii) Take the time derivative of (1) together with the result of part (i) above to show that $m_e^* = \hbar^2 \Big/ \frac{d^2\varepsilon}{dk^2}$. (6 marks)

(iv) What is the quantity $m_e^*$? Explain briefly, 3-4 lines only. (4 marks)

(v) What is the significance of $m_e^*$ for semiconductors and why is $m_e^*$ important technologically? (2 marks)

**Question 4** (22 Marks)

(a) The diagram at right shows schematically the energy bands of an n-type doped semiconductor. This semiconductor has electron effective mass $0.067m_e$ and dielectric constant 13.1. Assuming the bound donor electrons to be in Bohr orbits (hydrogenic impurity model),




(i) calculate the first donor ionization energy within the Bohr picture. (4 marks)

(ii) Given that the semiconductor exhibits a sharp increase in electrical conductivity when illuminated with radiation of wavelength $\lambda \leq 860$ nm find $E_d$ referenced to the top of the valence band. (4 marks)

(b) The valence band shown in the figure above may be taken to be parabolic and is described by $\varepsilon = -10^{-37}k^2$ J near to the band edge. An empty state at $\mathbf{k} = 10^9\hat{\mathbf{k}}_x$ m$^{-1}$ provides a mobile hole. For the mobile hole calculate, paying particular attention to the sign,

(i) the effective mass, (4 marks)
(ii) the energy, (4 marks)
(iii) the momentum, (3 marks)
(iv) the velocity. (3 marks)

**Question 5** (18 Marks)

Write brief notes (about 2-3 pages for each, including diagrams and any equations you include, but no more than this) on ***two only*** from the list of four topics given below.

Use simple diagrams and/or sketch graphs to illustrate your answers where appropriate ensuring that you label these and refer to them in your account.

(a) Crystalline solids. Your answer must discuss (but is not limited to) lattice and basis, bravais lattices and symmetry, unit cells, packing fraction. (9 marks)
(b) The BCS theory of superconductivity. Your account must include a description of the isotope effect, electron pairing and energy considerations, virtual phonon exchange and the superconducting energy gap. (9 marks)
(c) Effect of dopant concentration and temperature on the electrical conductivity of semiconductor materials. The relevant sketch graphs must be included in your answer. (9 marks)
(d) The Josephson effects. You may use information from the data sheet and refer to the diagram included below. (9 marks)