THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE/JULY 2007

PHYS3080

Solid State Physics

Time Allowed – 2 hours

Total number of questions - 5

Answer ALL questions

All questions are NOT of equal value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the Enrolment and Assessment Section: Calculators

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

PPS 64

Data and Formula Sheet

N.B. This is a generic PHYS3080 data/formula sheet and contains some additional information you may not necessarily need for this exam

$\mathbf{a}^* = \dfrac{2\pi(\mathbf{b}\mathrm{x}\mathbf{c})}{\mathbf{a}.(\mathbf{b}\mathrm{x}\mathbf{c})}$ and cyclic permutation of numerator

$e^x = 1 + x + \dfrac{x^2}{2}\ldots$ $\quad \displaystyle\int_0^{\Theta_D/T}\left(\frac{x^4 e^x dx}{(e^x-1)^2}\right) \cong \int_0^{\infty}\left(\frac{x^4 e^x dx}{(e^x-1)^2}\right) = \frac{4\pi^4}{15}$

$\dot{Q} = \dfrac{dQ}{dt} = \kappa A \dfrac{dT}{dx}$ $\quad C_v = 1/2\, k_B T\ \mathrm{mol}^{-1}$ per degree of freedom

$\kappa = \dfrac{1}{3}\bar{v} l C$ $\quad R = k_B/N_A$ $\quad E_{th} = k_B T$

$\varepsilon = E_g + \dfrac{\hbar^2 k^2}{2m_e}$ $\quad \varepsilon = -\dfrac{\hbar^2 k^2}{2m_h}$ $\quad E_n = -\dfrac{m_e^* e^4}{8h^2 n^2 \varepsilon_0{}^2}$ $\quad a = a_0 \varepsilon_r \left(\dfrac{m_e}{m_e^*}\right)$ $\quad a_0 = 0.053\ \mathrm{nm}$

$n_n p_n = n_i{}^2 = n_p p_p$ $\quad R_H = -\dfrac{1}{ne}$ $\quad n_i = p_i = (N_c N_v)^{1/2}\exp(-E_g/2k_B T)$

$np = (N_c N_v)\exp(-E_g/k_B T)$

$n \approx N_c \exp(-E_D/k_B T)$ for $k_B T << E_D$ $\quad p \approx N_v \exp(-E_A/k_B T)$ for $k_B T << E_A$

$\mathbf{F} = q(\mathbf{v}\mathrm{x}\mathbf{B})$ $\quad I = nAve$ $\quad \mathbf{v} = -\dfrac{e\tau}{m_e}\mathbf{E}$ $\quad \mathbf{J} = \sigma\mathbf{E}$ $\quad \sigma = ne\mu = \dfrac{ne^2\tau}{m}$ $\quad \mu = \dfrac{v_d}{E}$

$e = 1.6\mathrm{x}10^{-19}\ \mathrm{C}$ $\quad \varepsilon_0 = 8.854\mathrm{x}10^{-12}\ \mathrm{Fm}^{-1}$ $\quad N_A = 6.023\mathrm{x}10^{26}\ (\mathrm{kg.mol})^{-1}$

$h = 6.63\mathrm{x}10^{-34}\ \mathrm{Js}$ $\quad \hbar = 1.05\mathrm{x}10^{-34}\ \mathrm{Js}$ $\quad \hbar^2 = 1.11\mathrm{x}10^{-68}\ \mathrm{J^2s^2}$ $\quad \lambda_{visible} \sim 400-700\mathrm{nm}$

$v = \dfrac{1}{\hbar}\dfrac{d\varepsilon}{dk_x}$ $\quad m^* = \hbar^2 \Big/ \dfrac{d^2\varepsilon}{dk_x^2}$ $\quad j = j_0 \sin\left[\dfrac{2e}{\hbar}\left(V_0 t + \dfrac{v}{\omega}\sin(\omega t)\right) + \delta_0\right]$ $\quad V_0 = \dfrac{n\hbar\omega}{2e} = \dfrac{nh\nu}{2e}$

$n_{phonon} \sim \exp(-\Theta_D/T)$ $\quad \lambda_{phonon} \sim \exp(+\Theta_D/T)$

$k_F = \left(\dfrac{3\pi^2 N}{V}\right)^{1/3}$

$\xi_0 = \dfrac{\hbar v_F}{\pi\Delta(0)}$ $\quad V_0 = \dfrac{n\hbar\omega}{2e} = n\nu\Phi$

Question 1 (20 Marks)

(a) The Fermi-Dirac distribution function

$$f(\varepsilon)=\frac{1}{1+\exp\left(\frac{\varepsilon-\varepsilon_F}{k_B T}\right)}$$

gives the state occupation probability for electrons in a free electron metal.

(i) Define all symbols in this expression. **(3 marks)**

(ii) The total number of occupied electron states, $N(\varepsilon)$, in the energy range $\varepsilon \rightarrow \varepsilon + d\varepsilon$ is given by the product of the occupation probability $f(\varepsilon)$ with the density states function $g(\varepsilon)$. Sketch the form of the three quantities $N(\varepsilon)$, $f(\varepsilon)$, $g(\varepsilon)$ for a simple free electron metal. Indicate the situation for $T = 0K$ and $T_F >> T >> 0K$, where T_F is the Fermi temperature. (Put both curves on one plot or use a separate plot for each, as you prefer.) **(6 marks)**

(b) Give a concise explanation of the reason the observed electronic (i.e. the conduction electrons) contribution to the heat capacity of a metal is only a small fraction of that expected classically. Include a sketch illustrating your answer. **(4 marks)**

(c) The Fermi energy at 0K is given by $\varepsilon_{F,0} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$.

(i) Calculate $\varepsilon_{F,0}$ for aluminium metal (Al is *trivalent* with density $\rho_{Al} = 2.70 \times 10^3 \text{ kgm}^{-3}$ and atomic mass $26.98 \text{ kg}(\text{kmole})^{-1}$); give your answer in electron volts. **(3 marks)**

(ii) Determine the Fermi velocity and the de Broglie wavelength of an electron moving in aluminium at the Fermi energy. **(2 marks)**

(d) A particular sample of aluminium has drift velocity $v_d = 2.16 \text{ ms}^{-1}$ in an electric field $E = 500 \text{ Vm}^{-1}$. Estimate (i) the electron mobility; (ii) the relaxation (scattering) time. **(2 marks)**

Question 2 (22 Marks)

Consider a crystal for which the $\varepsilon - k$ relation is $\varepsilon = \varepsilon_1 + (\varepsilon_2 - \varepsilon_1)\sin^2(ak_x/2)$, with $\varepsilon_2, \varepsilon_1$ constants.

(a) Sketch and label the $\varepsilon - k$ dispersion relation for the first two Brillouin zones. **(4 marks)**

(b) Populate the band with one electron (i.e. assume one electron in the band), and, ignoring scattering of the electron and sketch and describe the behaviour of

(i) the effective mass, **(6 marks)**

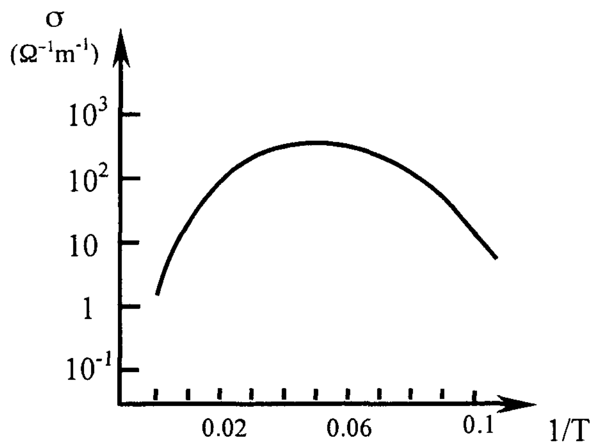
(ii) the electron velocity, **(6 marks)**

in a constant applied electric field E. Sketch each on a separate graph. Your sketch should include the range $-\pi/a < k_x \leq \pi/a$.

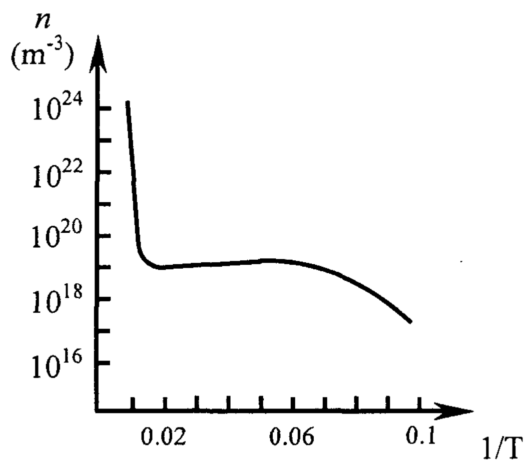
(c) Briefly describe the motion of the electron in this band when a constant electric field E is applied. **(6 marks)**

Question 3 (15 Marks)

(a) The electrical conductivity σ of a doped semiconductor specimen is measured over a wide temperature range with the following features being observed:
(i) At low temperatures σ rises with increasing temperature.
(ii) At intermediate temperatures σ falls with increasing temperature.
(iii) At high temperatures σ increases rapidly with increasing temperature.
The behaviour is illustrated on the graph at right; region (iii) is not shown.
Give the probable reasons for the behaviour in each temperature region. **(6 marks)**



(b) A Hall effect measurement is performed over a wide temperature range on the specimen discussed in part (a) above. The carrier concentration *n* in different temperature regions is determined from this measurement.
(i) From which temperature region would you estimate the band gap E_g? Give the reasons for your choice and state what you would plot graphically to find a value for E_g. **(5 marks)**
(ii) From which temperature region would you estimate the net donor concentration N_D-N_A? Give the reasons for your choice and estimate the value of N_D-N_A for this specimen. **(4 marks)**



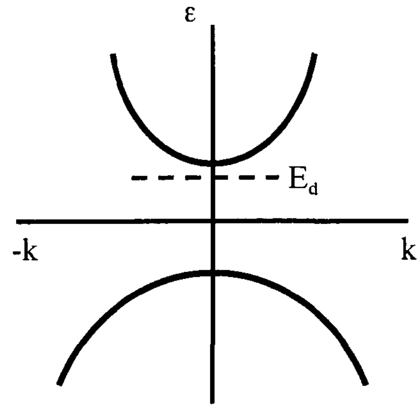
Question 4 (29 Marks)

(a) The diagram at right shows schematically the energy bands of an n-type doped semiconductor. This semiconductor has donor ionization energy $6.6\text{x}10^{-4}$ eV and dielectric constant 17.

(i) Calculate the electron effective mass for this semiconductor material. **(4 marks)**

(ii) Calculate the radius of the donor electron orbit. **(3 marks)**

(iii) Estimate the donor doping concentration, N_D, at which an impurity band will form. This will occur when adjacent donor orbits begin to overlap. **(4 marks)**



(b) Show by brief calculation (include all your working) and explain why,

(i) diamond ($Eg = 5.5$ eV) is a good insulator at 300K, **(3 marks)**

(ii) diamond is transparent in the optical region ($\lambda = 400\text{nm} - 700\text{nm}$) whereas silicon is opaque. **(4 marks)**

(c) Explain briefly with the aid of a simple sketch why gallium arsenide (GaAs) is suited to wide applications in optoelectronics whereas silicon is not. **(4 marks)**

(d) The Hall coefficient of a certain specimen of Si was found to be $R_H = -7.35\text{x}10^{-5}\ \text{m}^3\text{C}^{-1}$ from 100K to 400K.

(i) State whether this specimen is intrinsic or extrinsic at room temperature (300K), and, if extrinsic, state the type of doping (is it n-type or p-type doped?). **(3 marks)**

(ii) The electrical conductivity of this Si specimen was found to be $200\ (\Omega\text{m})^{-1}$. Calculate the density and the mobility of the charge carriers. **(4 marks)**

Question 5 (14 Marks)

Write brief notes (about 2-3 pages for each, including diagrams and any equations you include, but no more than this) on ***two only*** from the list of four topics given below. Use simple diagrams and/or sketch graphs to illustrate your answers where appropriate and refer to these in your account.

(a) The BCS theory of superconductivity. (**7 marks**)
(b) Effect of dopant concentration and temperature on the electrical conductivity of semiconductor materials. (**7 marks**)
(c) The Josephson effects. (**7 marks**)
(d) The Kronig-Penney model of energy bands. (**7 marks**)