THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE/JULY 2006

**PHYS3080**

**Solid State Physics**

Time Allowed – 2 hours

Total number of questions - 5

Answer ALL questions

All questions are NOT of equal value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the

Enrolment and Assessment Section: Calculators

Answers must be written in ink. Except where they

are expressly required, pencils may only be used

for drawing, sketching or graphical work

**Data and Formula Sheet**

$$\dot{Q} = \frac{dQ}{dt} = \kappa A \frac{dT}{dx}$$

$$\kappa = \frac{1}{3}\bar{v} l C$$

$$\varepsilon = E_g + \frac{\hbar^2 k^2}{2m_e} \qquad \varepsilon = -\frac{\hbar^2 k^2}{2m_h} \qquad E_n = -\frac{m_e^* e^4}{2\varepsilon^2 \hbar^2 n^2 (4\pi\varepsilon_0)^2} \qquad n_n p_n = n_i^2 = n_p p_p$$

$$\mathbf{F} = q(\mathbf{v} x \mathbf{B}) \qquad I = nAve \qquad \mathbf{v} = -\frac{e\tau}{m_e}\mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E} \qquad \sigma = ne\mu$$

$$\varepsilon_0 = 8.854 x 10^{-12}\ \mathrm{Fm^{-1}}$$

$$N_A = 6.023 x 10^{26}\ (\mathrm{kg.mol})^{-1}$$

$$h = 6.63 x 10^{-34}\ \mathrm{Js} \qquad \hbar = 1.05 x 10^{-34}\ \mathrm{Js} \qquad \hbar^2 = 1.11 x 10^{-68}\ \mathrm{J^2 s^2}$$

$$j = j_0 \sin\left[\frac{2e}{\hbar}\left(V_0 t + \frac{v}{\omega}\sin(\omega t)\right) + \delta_0\right],\ V_0 = \frac{n\hbar\omega}{2e} = \frac{nh\nu}{2e}$$

$$n_{phonon} \sim \exp(-\Theta_D/T)$$

$$\lambda_{phonon} \sim \exp(+\Theta_D/T)$$

$$e = 1.6 x 10^{-19}\ \mathrm{C} \qquad k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$$

$$\xi_0 = \frac{\hbar v_F}{\pi\Delta(0)}$$

$$V_0 = \frac{n\hbar\omega}{2e} = n\nu\Phi$$

**Question 1** (20 Marks)
In a solid the mean energy per oscillator (or phonon energy) is given by

$$\bar{\varepsilon} = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\left(e^{\hbar\omega/k_B T} - 1\right)}$$

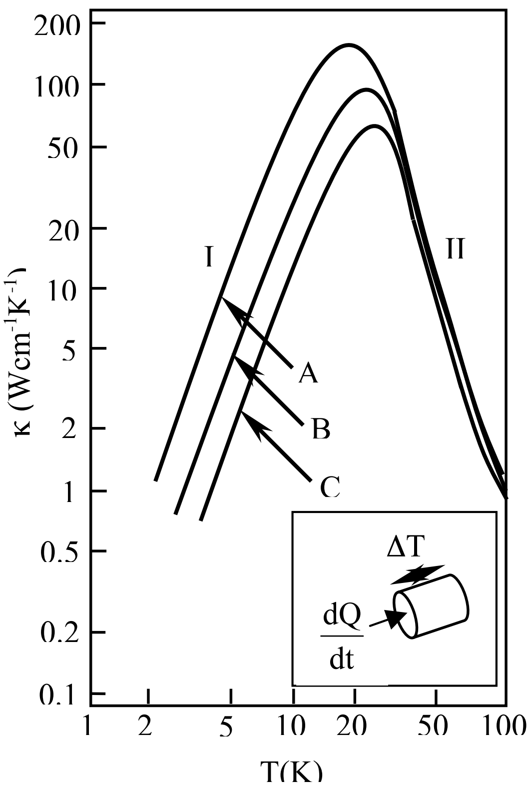
(i) What does the first term on the right, $\frac{1}{2}\hbar\omega$, represent?

(ii) Explain briefly what the factors $\hbar\omega$ and $\frac{1}{\left(e^{\hbar\omega/k_B T} - 1\right)}$ in the second term represent.

(iii) Use this expression to show that, in thermal equilibrium at temperature T, the energy of a sufficiently long wavelength mode is $k_B T$.
(iv) Estimate the number of modes that will be excited at temperatures $T << \theta_D$.
(v) Thus show that for $T << \theta_D$ the heat capacity is given approximately by $C_V \sim Nk_B(T/\theta_D)^3$

**Question 2** (18 Marks)

The diagram below shows thermal conductivity data as a function of temperature for three pure, crystalline specimens of LiF labeled A, B and C. The inset (bottom right) indicates schematically how the measurements were performed: a constant heat current $\dot{Q}$ into the crystal face of area A produced a measured temperature difference $\Delta T$ along the specimen. In region I the thermal conductivity follows a $T^3$ law.




(i) Assuming A, B and C to have identical chemical composition, morphology and purity, what is the probable cause of the difference seen in the data for the three specimens below T~20K; explain briefly (no more than 4 or 5 lines).

(ii) The kinetic theory (for classical gases) expression for thermal conductivity $\kappa$ must be modified when applied to solids and for different temperature ranges. What are the physical parameters and/or mechanisms determining $\kappa$ in regions I and II (marked on graph) respectively? Provide a brief (no more than 4-5 lines) explanation for each.

(iii) For a LiF specimen in the form of a right circular cylinder, diameter 4 mm x length 1cm, *estimate* the heat current required to produce a temperature difference of 0.05 K along the length of the crystal at T~50K.

**Question 3** (25 Marks)

(a) For electrons moving in the energy band of a one-dimensional chain of atoms we have the two important results

(1) $v = \dfrac{d\omega}{dk} = \dfrac{1}{\hbar}\dfrac{d\varepsilon}{dk}$ velocity of an electron wavepacket

and

(2) $\delta\varepsilon = -eE\delta x$ motion of electron wavepacket in a uniform, static electric field E.

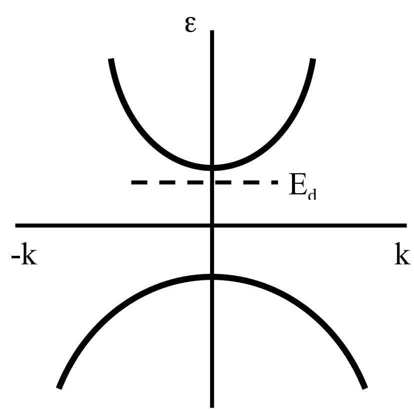
(i) Use results (1) and (2) to show that $\hbar\dfrac{dk}{dt} = -eE$; show your working.

(ii) What is the quantity $\hbar k$? Explain briefly, 3-4 lines only.

(iii) Take the time derivative of (1) together with the result of part (i) to show that $m_e = \hbar^2 \Big/ \dfrac{d^2\varepsilon}{dk^2}$.

(iv) What is the quantity $m_e$? Explain briefly, 3-4 lines only.

(b) The diagram at right shows schematically the energy bands of an n-type doped semiconductor. This semiconductor has electron effective mass $0.067m_e$ and dielectric constant 13.1.

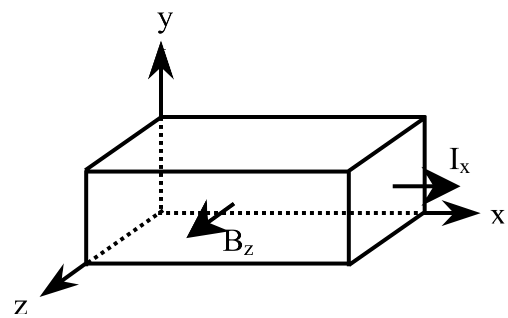



(i) Assuming the bound donor electrons to be in Bohr orbits (hydrogenic impurity model),
(ii) Calculate the first donor ionization energy within the Bohr picture.
(iii) Given that the semiconductor exhibits a sharp increase in electrical conductivity when illuminated with radiation of wavelength $\lambda \leq 860$ nm find $E_d$ referenced to the top of the valence band.

(c) The valence band shown in the figure above may be taken to be parabolic and is described by $\varepsilon = -10^{-37}k^2$ J near to the band edge. An empty state at $\mathbf{k} = 10^9\hat{\mathbf{k}}_x$ m$^{-1}$ provides a mobile hole. For the mobile hole calculate, paying particular attention to the sign,
(i) the effective mass,
(ii) the energy,
(iii) the momentum,
(iv) the velocity.

**Question 4** (19 Marks)

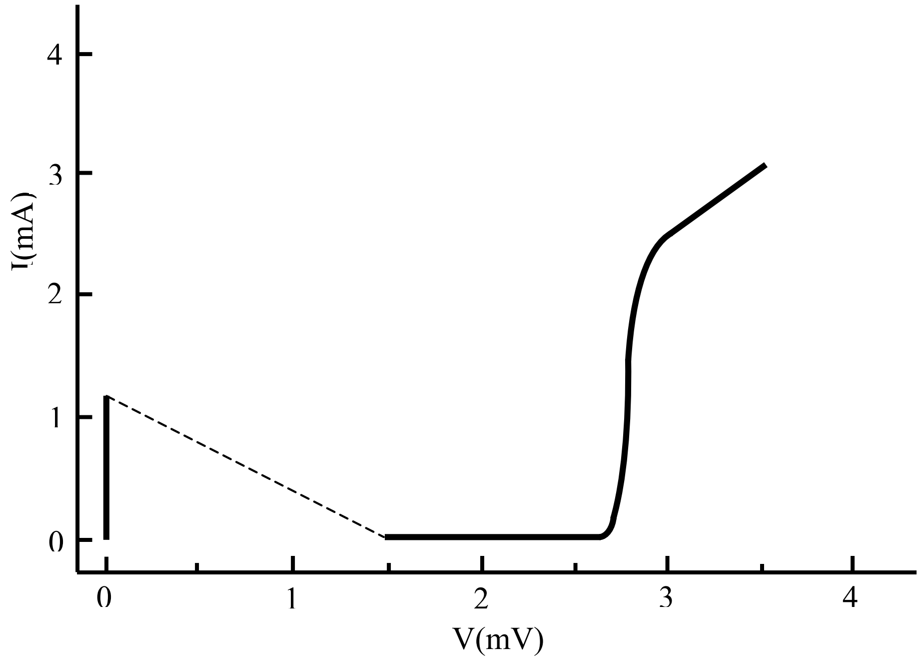
The figure at right shows schematically a conducting specimen (not to scale) for observation of the Hall effect. Assume the majority carriers are electrons. The specimen carries *conventional* current $\mathbf{I} = I_x$. An applied magnetic induction $\mathbf{B} = B_z$ is used to observe the Hall effect. The specimen has dimensions length $l$ (along x), width w (in the y direction) and thickness t (in z the direction).




(a) Reproduce this diagram as a sketch in your answer book and mark on your sketch,
(i) the velocity **v** of the charge carriers,
(ii) the direction of the force $\mathbf{F}_m$ on the charge carriers due to $\mathbf{B} = B_z$,
(iii) the resulting accumulations of charges caused by this force (show this by marking a few charges of each sign in the appropriate region of your sketch),
(iv) the direction of the Hall electric field.
(v) In equilibrium $\mathbf{F}_m$ is balanced by the force on charge carriers arising from the Hall field. Use this equilibrium condition to derive an expression for the charge carrier concentration $n$ in terms of charge q, $B_z$, $I_x$, Hall voltage $V_H$ and the specimen dimensions.
(b) A specimen of n-type silicon at room temperature has resistivity $0.5\ \Omega\text{m}$ and mobility $0.15\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$. For this specimen,
(i) calculate the donor concentration,
(ii) calculate the equilibrium minority carrier concentration at room temperature.

**Question 5** (18 Marks)

(a)




The diagram above shows I-V data for a Pb-oxide-Pb junction at $T = 1.2\ \mathrm{K}$.

(i) Which physical process causes the finite current observed at $V = 0$ (i.e. the zero-voltage current feature)? Provide 1-2 lines of description and name this phenomenon.

(ii) Which physical process occurs to provide the current flow observed at finite voltages? 1-2 lines of description only.

(iii) Use the graph above to obtain a value for the superconducting energy gap Δ. (Remember to state the units of your answer.)

(iv) Use this information to estimate the BCS coherence length for lead. Take $k_F = 1.57\text{x}10^{10}\ \mathrm{m}^{-1}$

(v) For superconducting lead, make an order-of-magnitude estimate of the number of electrons to be found between the two electrons of any Cooper pair.

(b) The graph below shows I-V data for a niobium point contact junction in a microwave field at frequency $f_{\mu wave} = 72\ \mathrm{GHz}$. The temperature is $T = 4.2\ \mathrm{K}$. From the data, estimate the value of the flux quantum; show your working and give a brief (1-2 lines only) explanation of your procedure.