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THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – NOVEMBER 2009

PHYS3060 – ADVANCED OPTICS

Time allowed – 2 hours

Total number of questions – 4

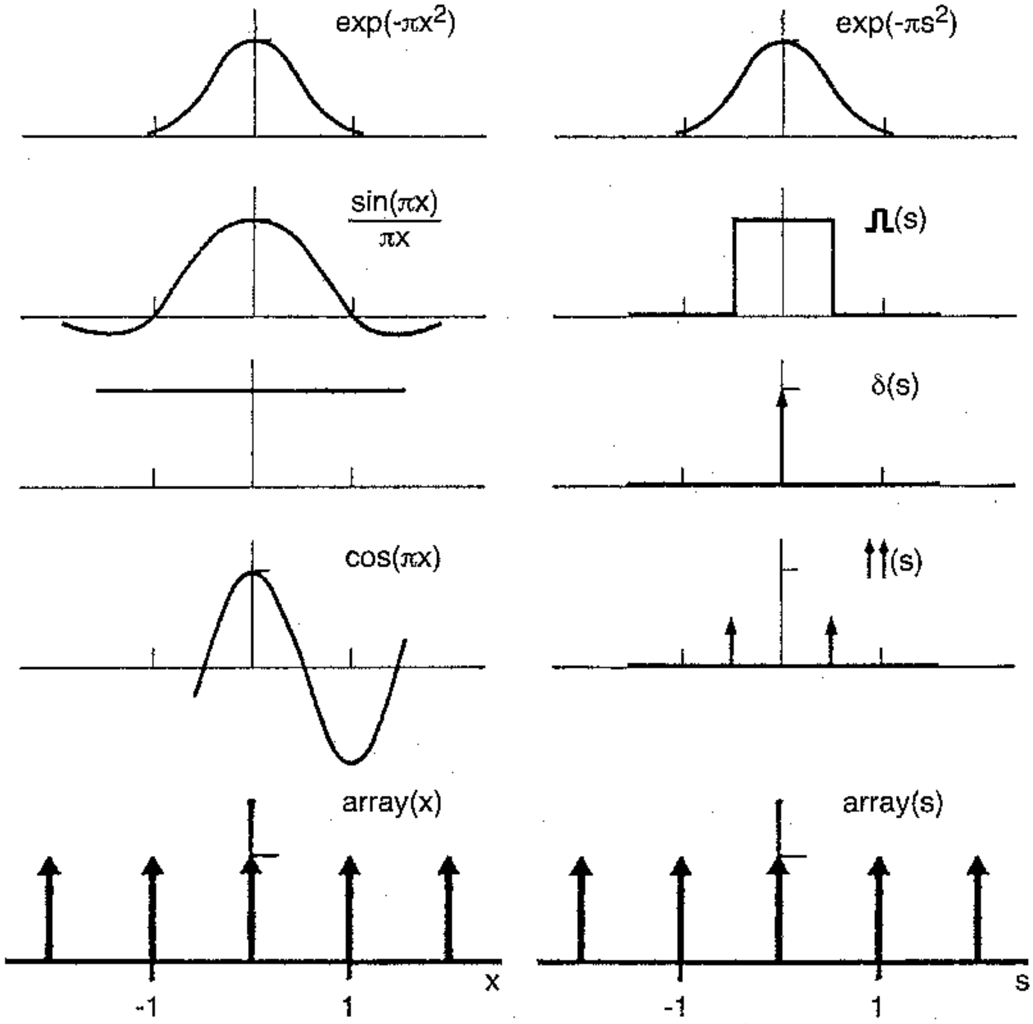
Attempt **ALL** questions

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

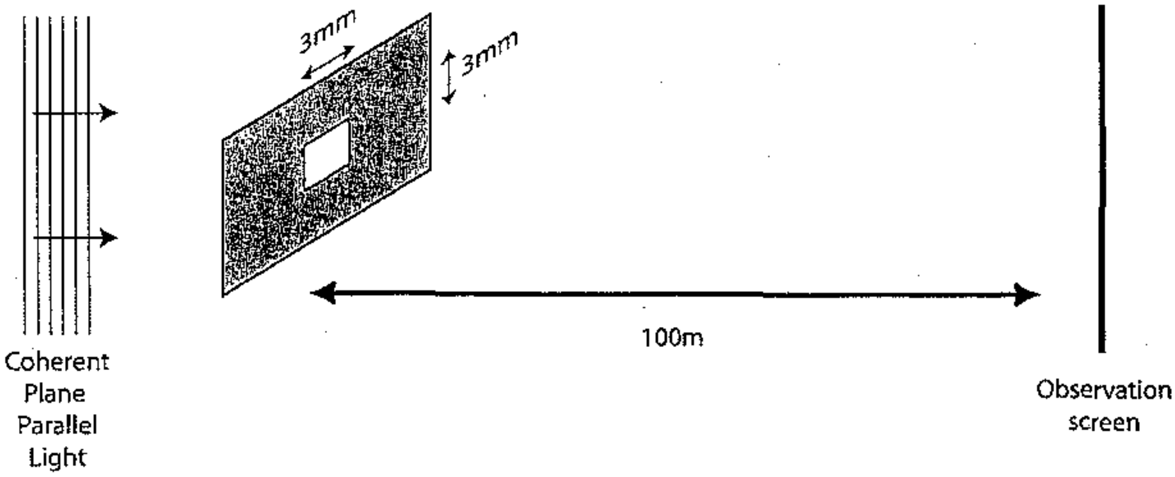


Theorems of the Fourier Transform

Theorem	$f(x)$	$F(s)$
Similarity	$f(ax)$	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$
Linearity	$\lambda f(x)+\mu g(x)$	$\lambda F(s)+\mu G(s)$
Shift	$f(x-a)$	$e^{-2\pi i a s}F(s)$
Convolution	$f(x)\otimes g(x)$	$F(s)\bullet G(s)$

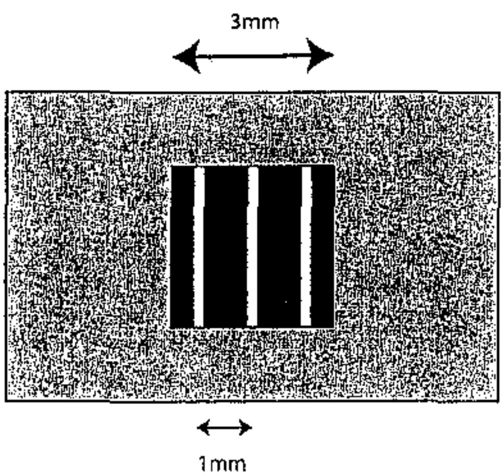
Question 1

A small square hole is cut in a large, opaque sheet of metal. The dimensions of the square hole are 3 mm by 3 mm. It is illuminated by a coherent source of monochromatic light with a wavelength of 500 nm. An observation screen is set up 100 m away from the hole, so as to observe the diffraction pattern.



(i) Show that Fraunhofer theory can be used to calculate the diffraction pattern for the situation described above.
(ii) Using Fourier theory (or otherwise) derive an expression for the diffracted field produced on the observation screen.
(iii) Sketch the diffraction intensity pattern produced on the observation screen, marking positions of zeros in a real-space distance (as they would be seen by an observer on the screen).

A second mask is cut from another large, opaque sheet of metal. In this mask, three very long, narrow, parallel slits are cut so that the inter-slit separation is 1 mm. This second mask is placed over the first mask, so that the middle slit is centred in the square and the slits are parallel to the edge of the square.



(iv) Derive an expression for the new diffraction pattern (field or intensity) produced by the combined masks.

(v) Sketch the diffracted intensity distribution produced on the screen, marking the positions of maxima and zeros in a real space (as they would be measured by an observer).

The original 500 nm wavelength light source is replaced with a new source, which has a wavelength of 700 nm.

(vi) How will the diffraction pattern change with the new source?

Question 2

Answer **TWO** of the following questions. Use words, pictures and/or equations to illustrate your answer. Give numerical examples wherever possible.

(i) The Fourier transform of an object decomposes it into spatial frequencies. Fraunhofer diffraction theory shows that the diffracted field produced by mask which is small compared to the distance between the mask and the observation plane is given by a Fourier transform. Explain why the diffraction pattern is related to the spatial frequencies in the diffraction mask. Give examples.

(ii) Explain the mathematical operation of convolution. Illustrate your explanation with examples. Explain the meaning of the convolution theorem.

(iii) Sketch the derivation of Kirchhoff's scalar theory of diffraction which starts with Maxwell's equations of electromagnetism and results in a diffraction equation that has the same form as the Huygens-Fresnel diffraction equation.

(iv) Explain transverse (spatial) and longitudinal (temporal) coherence. Give examples. Show how these concepts are related to elementary quantum mechanics.

(v) Matrix methods are used to compute geometric optical effects in ray tracing programs. Explain how these methods work. Give specific examples.

(vi) Explain how a Fresnel lens or zone plate focuses light. Describe the properties of Fresnel lenses.

(vii) Prove one of the Fourier relations given in the transform pairs as shown schematically at the front of this exam paper.

(viii) Prove one of the Fourier theorems given in the information sheet at the front of this exam paper.

(ix) Explain how a diffraction grating works as a wavelength dispersive element. Show that the wavelength resolution of a diffraction grating is just dependent on the number of slits in the grating.

(x) Sketch the derivation of the van Cittert-Zernike theorem.

Question 3

The Huygens-Fresnel equation for a scalar field $\Psi(\zeta,\eta)$ produced by a diffracting aperture with transmission function, f(x,y) can be expressed:

$$\psi(\zeta,\eta) = \iint_{aperture} f(x,y)\frac{e^{ikR}}{R}\kappa(\chi)\,dxdy$$

(i) Explain the physical significance of the term: $\frac{e^{ikR}}{R}$.

(ii) Explain the physical significance of the term: $\kappa(\chi)$.

(iii) What is the justification for including the term $\kappa(\chi)$?

In the case of far-field or Fraunhofer diffraction, the Huygens-Fresnel equation is simplified so that the diffracted field is given by the Fourier transform of the diffraction aperture transmission function f(x,y) as per the following equation:

$$\psi(u,v) = \frac{\kappa(\chi)e^{ikR_o}}{R_o}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-2\pi i(ux+yv)}dxdy$$

(iv) What is the physical meaning of the Fourier variables (u, v) used to describe the diffracted field?

(v) How are the Fourier variables (u, v) related to the original variables (ζ, η) that represent distances in the diffraction plane?

(vi) How are the Fourier variables related to diffraction angles in two dimensions (θ, ϕ)?

The condition for Fraunhofer or far-field diffraction is:

$$\left(\frac{x^2+y^2}{2\lambda R_o}\right) \ll 1$$

(vii) Explain the meaning of this condition.

(viii) What is the origin of this condition?

Finally, the diffraction pattern produced using a lens is also given by the Fourier transform of the transmission function f(x,y) of a diffraction mask. In this case, there is no need for the Fraunhofer condition.

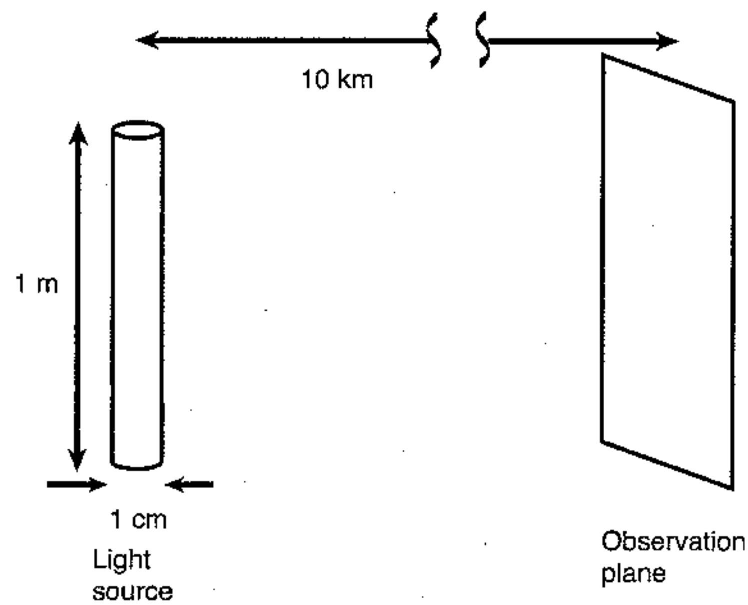
(ix) Explain why the diffraction equation for a lens does not require the Fraunhofer condition?

Question 4
Part A

(i) Describe what is meant by transverse or lateral coherence.
(ii) Describe what is meant by longitudinal coherence.
(iii) How are these two measures of coherence related to the physical properties of a thermal light source?
(iv) For a given thermal source, describe how one could experimentally determine the two measures of coherence. Use diagrams to describe the measurement apparatus.

Part B

An ultra powerful fluorescent light tube can be approximated as a rectangular, incoherent source of dimensions: 1 meter by 1 cm. A wavelength filter is placed directly in front of the tube to limit the wavelength of the emitted light to a narrow spectral range centred on 500nm. The quasi-monochromatic light from this tube propagates to a distant observation plane 10 km away.



(v) Use the van Cittert-Zernike theorem to obtain an analytic expression for the complex degree of coherence, γ_{12}, of light reaching the observation plane. (assume that far field conditions apply).
(vi) Sketch the complex degree of coherence as a function of the separation between two points in the observation plane.
(vii) What is the meaning of this function and what implications does it have for light in the observation plane?