THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – NOVEMBER 2008

PHYS3060 – ADVANCED OPTICS

Time allowed – 2 hours

Total number of questions – 4

Attempt **ALL** questions

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the

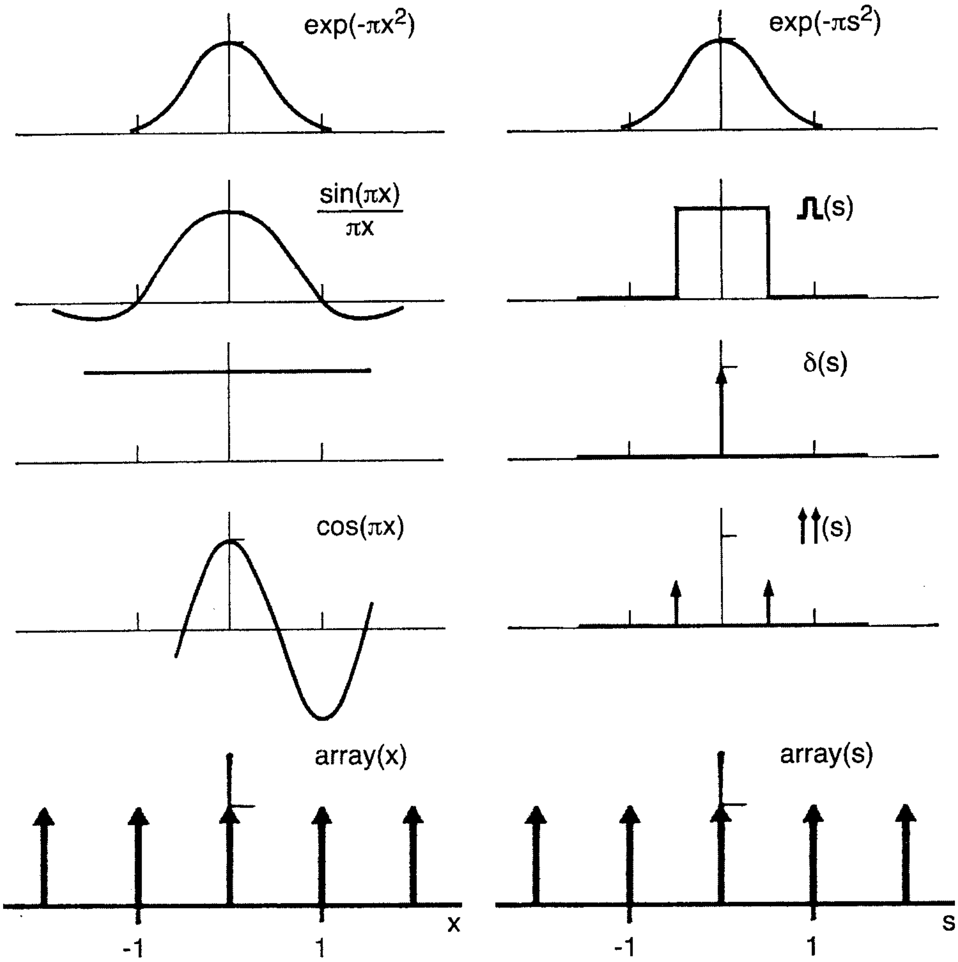
Enrolments and Assessments Section: Calculators

Answers must be written in ink. Except where they

are expressly required, pencils may only be used for

drawing, sketching or graphical work.

p3



Theorems of the Fourier Transform

Theorem	$f(x)$	$F(s)$
Similarity	$f(ax)$	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$
Linearity	$\lambda f(x)+\mu g(x)$	$\lambda F(s)+\mu G(s)$
Shift	$f(x-a)$	$e^{-2\pi i a s}F(s)$
Convolution	$f(x)\otimes g(x)$	$F(s)\bullet G(s)$

Question 1

Part A

You are given the following components and asked to construct a system to produce two-dimensional optical Fourier transforms of objects photographed onto slides:

He-Ne laser (wavelength 632.8nm)
Optical bench rail with holders
Large variable iris aperture (radius varies from 2mm to 2cm)
2 large lenses (f=20cm)
Microscope objective lens (f=2mm)
CCD optical detector plus computer and software
A set of pinhole foils with different diameter holes
Several 2D translation stages with slide/object holders

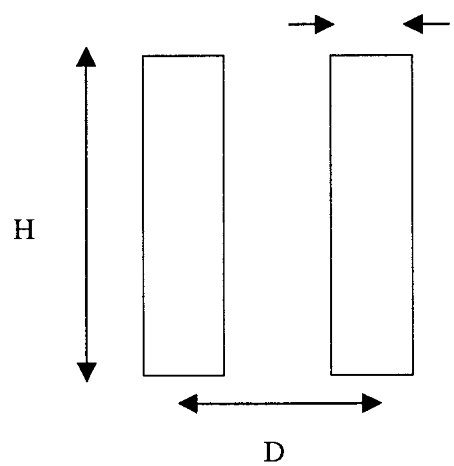
(i) Draw a diagram of your optical Fourier transformer. Mark the separations between key components where these distances are critical.
(ii) Explain the role of the various components in your system.

The objects whose Fourier transforms you wish to determine are all approximately 1cm in diameter on their respective transparencies.

(iii) Discuss how you would decide on the size of the pinhole that would be most appropriate for you system. What are the key considerations?

Part B

A simple opaque mask with three rectangular slits cut into it is placed on the optical bench. Each pair of slits are separated by a centre-to-centre distance D in a direction parallel to their widths and each slit is W wide and H high.



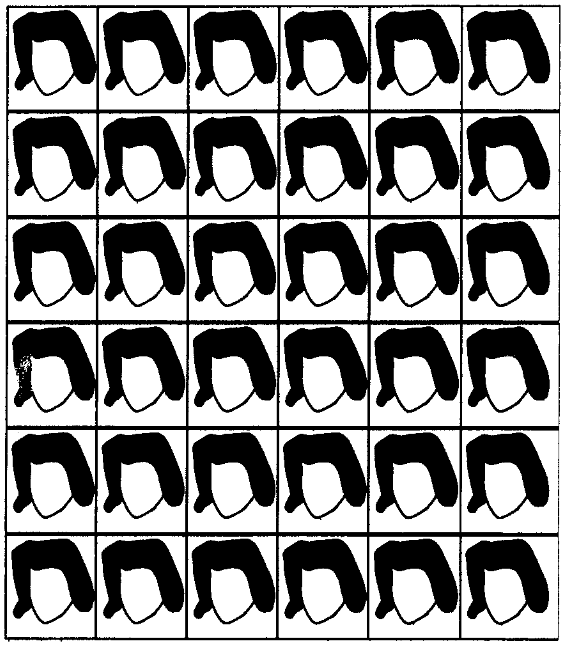
(iv) Determine a mathematical expression for the Fraunhofer diffracted field produced by the mask in terms of diffraction space variables: s = (u,v).
(v) Hence determine an expression for the diffracted intensity.

(vi) Sketch the diffracted intensity function marking all zeros and indicating regions of intense diffraction.

Part C

More complex masks include reproductions of various artworks.

(vii) Describe the properties of the diffraction pattern obtained from the image shown below.



Question 2

Answer **TWO** of the following questions. Use words, pictures and/or equations to illustrate your answer. Give numerical examples wherever possible.

(i) The Fourier transform of an object decomposes it into spatial frequencies. Fraunhofer diffraction theory shows that the diffracted field produced by mask which is small compared to the distance between the mask and the observation plane is given by a Fourier transform. Explain why the diffraction pattern is related to the spatial frequencies in the diffraction mask. Give examples.

(ii) Explain the mathematical operation of convolution. Illustrate your explanation with examples. Explain the meaning of the convolution theorem.

(iii) Sketch the derivation of Kirchhoff's scalar theory of diffraction which starts with Maxwell's equations of electromagnetism and results in a diffraction equation that has the same form as the Huygens-Fresnel diffraction equation.

(iv) Explain transverse (spatial) and longitudinal (temporal) coherence. Give examples. Show how these concepts are related to elementary quantum mechanics.

(v) Explain how a Fresnel lens or zone plate focuses light. Describe the properties of Fresnel lenses.

(vi) Prove one of the Fourier relations given in the transform pairs as shown schematically at the front of this exam paper.

(vii) Prove one of the Fourier theorems given in the information sheet at the front of this exam paper.

(viii) Explain how a diffraction grating works as a wavelength dispersive element. Show that the wavelength resolution of a diffraction grating is just dependent on the number of slits in the grating.

(ix) Sketch the derivation of the van Cittert-Zernike theorem.

Question 3

The Huygens-Fresnel equation for a scalar field $\Psi(\zeta,\eta)$ produced by a diffracting aperture with transmission function, f(x,y) can be expressed:

$$\psi(\zeta,\eta) = \iint_{aperture} f(x,y)\frac{e^{ikR}}{R}\kappa(\chi)dxdy$$

(i) Explain the physical significance of the term: $\frac{e^{ikR}}{R}$.

(ii) Explain the physical significance of the term: $\kappa(\chi)$.

(iii) What is the justification for including the term $\kappa(\chi)$?

In the case of far-field or Fraunhofer diffraction, the Huygens-Fresnel equation is simplified so that the diffracted field is given by the Fourier transform of the diffraction aperture transmission function f(x,y) as per the following equation:

$$\psi(u,v) = \frac{\kappa(\chi)e^{ikR_o}}{R_o}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-2\pi i(ux+yv)}dxdy$$

(iv) What is the physical meaning of the Fourier variables (u, v) used to describe the diffracted field?

(v) How are the Fourier variables (u, v) related to the original variables (ζ, η) which represent distances in the diffraction plane?

(vi) How are the Fourier variables related to diffraction angles in two dimensions (θ, ϕ)?

The condition for Fraunhofer or far-field diffraction is:

$$\left(\frac{x^2+y^2}{2\lambda R_o}\right) << 1$$

(vii) Explain the meaning of this condition.

(viii) What is the origin of this condition?

Finally, the diffraction pattern produced using a lens is also given by the Fourier transform of the transmission function f(x,y) of a diffraction mask. In this case, there is no need for the Fraunhofer condition.

(ix) Explain why the diffraction equation for a lens does not require the Fraunhofer condition?

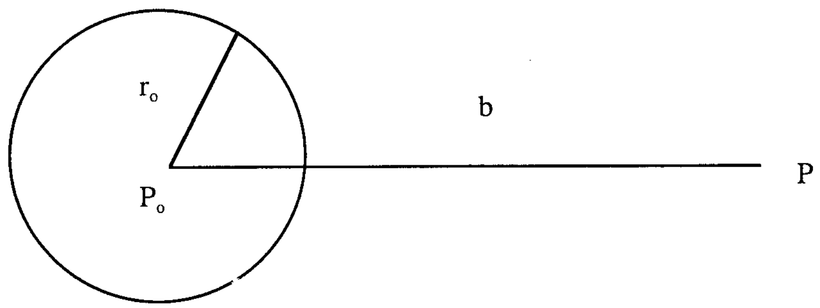
Question 4

Part A

Fresnel theory for near-field diffraction from a circularly symmetric mask uses the Fresnel Zone construction.

(i) What is a Fresnel Zone? Describe its properties and how it is constructed.

The diagram below shows a point source at P_o, an imaginary sphere centred on P_o of radius r_o whose surface is divided into Fresnel zones, and a point P where the optical field is to be calculated.



The field arriving at P from the jth Fresnel zone is given by:

$$\phi_j = \frac{Ae^{ik(r_o+b)}}{r_o+b}(-2i\lambda Q)(-1)^j \kappa_j$$

(ii) How was this equation derived? What were the approximations used in the derivation?

(iii) How will the field arriving at P from the (j+1)th zone compare with that arriving at P from the jth zone?

(iv) Use the zone construction (or otherwise) to explain Poisson's spot – the bright spot observed in the centre of the geometric shadow produced by an opaque disc.

Part B

(v) How is the visibility of interference fringes defined so as to measure coherence properties of a thermal source?

(vi) Explain what is meant by the complex degree of coherence, γ_{12} or $\gamma(\mathbf{r}_1, \mathbf{r}_2, t)$?

(vii) Discuss the van Cittert-Zernike theorem and how it relates the complex degree of coherence to the distance, size and shape of a thermal light source.

(viii) Use the van Cittert-Zernike theorem to compute the coherence properties of the light field produced by a car headlamp at a distance of 100m (assuming that the filament is a rectangular thermal source (1mm x 5mm).