THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – NOVEMBER 2007

PHYS3060 – ADVANCED OPTICS

Time allowed – 2 Hours

Total number of questions – 4

Attempt **ALL** questions

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the

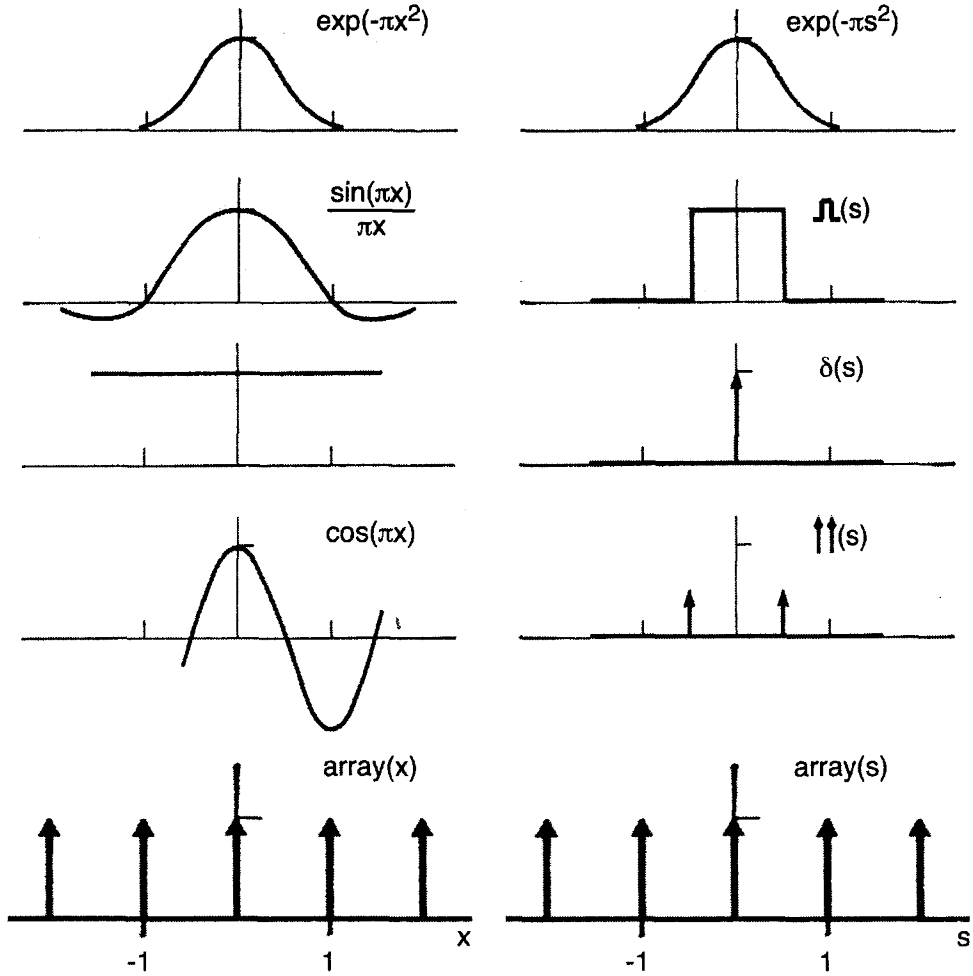
Enrolments and Assessments Section: Calculators

Answers must be written in ink. Except where they

are expressly required, pencils may only be used for

drawing, sketching or graphical work.

PPS



Theorems of the Fourier Transform

Theorem	$f(x)$	$F(s)$
Similarity	$f(ax)$	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$
Linearity	$\lambda f(x)+\mu g(x)$	$\lambda F(s)+\mu G(s)$
Shift	$f(x-a)$	$e^{-2\pi ias}F(s)$
Convolution	$f(x)\otimes g(x)$	$F(s)\bullet G(s)$

Question 1

A diffraction mask is created by drilling two tiny pinholes into an opaque plate. The pinholes are separated by 1 mm in the horizontal direction. The mask is illuminated by coherent monochromatic plane parallel light with a wavelength of 500nm. An observation screen is placed 100 metres from the mask.

(i) Show that the Fraunhofer condition is satisfied for this diffraction experiment.

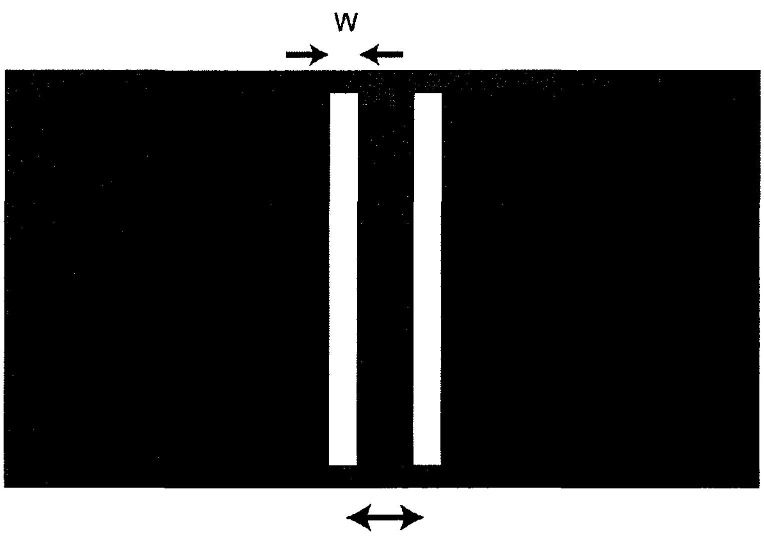
(ii) Write an expression that describes the transmission function of the diffraction mask.

(iii) Using Fourier theory (or otherwise), derive the equation for the diffracted field in the plane of the observation screen 100 metres from the mask.

(iv) Sketch a graph representing the diffracted field in the plane of the observation screen using a distance scale in the appropriate metric unit (e.g. mm, cm, m, km etc). Carefully mark the positions of all zeros, maxima and minima of the field.

(v) Sketch the diffracted intensity pattern in the observation plane using the same scale as for the diffracted field. Mark the positions of zeros and maxima.

The original mask is altered by replacing the two pinholes by a pair of very long parallel slots, each of width W, where W is less than half the 1 mm separation between the centres of the slots.



(vi) Derive an expression for the diffracted intensity observed on the screen 100 metres from the new diffraction mask.

(vii) Sketch the intensity pattern seen on the observation screen.

The mask is altered again. This time the slot width W is enlarged so that it is just under 1 mm.

(viii) Sketch the intensity pattern seen on the observation screen.

(ix) Comment on the result.

Question 2

Answer **TWO** of the following questions. Use words, pictures and/or equations to illustrate your answer. Give numerical examples wherever possible.

(i) Describe how Fresnel or near field diffraction can be thought of as a convolution process. Give examples.

(ii) Explain the meaning of spatial frequencies. Give examples. Explain how spatial frequencies in an object are intimately linked to diffraction effects produced by the object.

(iii) Explain the difference between temporal and spatial (or transverse) coherence. Give examples of these two types of coherence. Explain how each of these coherence properties is related to the properties of a light source. Give numerical examples.

(iv) Explain the origin of Poisson's spot (bright spot in the centre of the geometric shadow produced by an opaque circular disc). Why is it not easily observed?

(v) Outline how the Kirchhoff-Fresnel equation can be derived from Maxwell's equations of electromagnetism.

(vi) The Fourier transform of an object of finite dimension contains redundant information. Show how the Fourier transform of such an object can be sampled so that the original object can be recreated via an inverse Fourier transform without loss of resolution. Discuss the implications of these findings.

(vii) Explain how matrix methods can be used to solve practical problems in geometric optics. Give examples of how the methods work.

(viii) Explain why the van Cittert-Zernike theorem for determining the complex degree of coherence has the same form as the diffraction equation. Explain the parallels between a diffraction problem and a coherence problem using an example.

Question 3

The Huygens-Fresnel theory of diffraction is based on two principles:

1. each point on a wavefront acts as a source of secondary wavelets
2. these 2° wavelets produce a new wavefront by mutual interference.

(i) Draw a diagram to explain how the Huygens-Fresnel construction works.

This construction results in the Huygens-Fresnel diffraction equation:

$$\psi(\zeta,\eta) = \iint_{aperture} f(x,y)\frac{e^{ikR}}{R}\kappa(\chi)dxdy$$

where f(x,y) is the transmission function of a diffraction aperture and $\psi(\zeta,\eta)$ is the diffracted field.

(ii) Explain the meaning of the function $\kappa(\chi)$ in the Huygens-Fresnel equation.

A more fundamental diffraction theory is the Kirchhoff Scalar theory, which is based on Maxwell's electromagnetic equations. In this theory, an expression for a field at point P, $\phi(P)$, arising from a diffraction aperture illuminated by a point source Is given by the Kirchhoff-Fresenl equation:

$$\phi(P) = \frac{-i}{\lambda}\iint_{Aperture} \frac{Ae^{ik\rho}}{\rho}\frac{e^{ikr}}{r}\left(\frac{\hat{n}\bullet\hat{\rho}-\hat{n}\bullet\hat{r}}{2}\right)dS$$

(iii) Explain how the terms in the Huygens-Fresnel equation correspond to terms in the Kirchhoff-Fresnel equation.

(iv) What is the physical meaning of the additional terms in the Kirchhoff-Fresnel equation that are absent in the Huygens-Fresnel equation.

In the case of far field diffraction, the diffraction equations can be used to show that the diffracted field is related to the aperture transmission function by a Fourier transform. This derivation results in the Fraunhofer condition:

$$\frac{r_{\max}^2}{2\lambda R_o} \ll 1$$

(v) Explain the meaning of the terms in this equation.

(vi) What is the origin of this condition in terms of the derivation of the Fraunhofer diffraction equation?

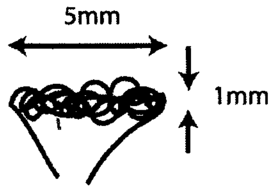
(vii) Explain why this condition is not necessary when a lens is used to produce a diffracted field in its back focal plane.

Question 4

(i) What is meant by the complex degree of coherence, γ_{12} or $\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$?

(ii) Explain the relationship between the complex degree of coherence and the visibility of interference fringes? Give an example to illustrate this connection.

The van Cittert-Zernike theorem can be used to compute the complex degree of coherence for distant thermal sources. Consider a car headlamp as a thermal source at a distance of 500m. Approximate the lamp filament as a rectangular source, 5mm long and 1mm wide.



(iii) Derive an expression for the complex degree of coherence of 500nm light produced by the car headlamp at a distance of 500m.

(iv) Plot the complex degree of coherence as a function of separation of two points.

Two pinholes are placed in a screen 500 m from the car headlamp with a filter to remove all light except for that with a wavelength of 500 nm. The pinholes are oriented so that the line joining them is perpendicular to the long axis of the headlamp filament. The distance between the two pinholes can be varied. The interference pattern produced is observed on a second screen.

(v) Describe the interference pattern observed as the distance between the pinholes is varied.

(vi) Sketch a series of interference patterns to illustrate key features of the complex degree of coherence.