Midsession test

Duration: 50 minutes
University-approved calculators may be used.

Question 1 (15 marks)
The radial Schrödinger equation for a central potential in spherical coordinates, letting $\psi(r,\theta,\phi) = \frac{\chi_l(r)}{r} Y_{lm}(\theta,\phi)$, is:

$$\frac{d^2\chi_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right)\chi_l(r) = 0 .$$

(a) Estimate the depth of the proton-neutron potential approximated by the spherical square well. It is known from experiment that the binding energy is close to zero (deuteron), nucleon mass $m \approx 940$ MeV, and the radius of the potential $R \approx 2$ fm.

(b) Compare your answer for (a) with the Coulomb interaction of two protons at a distance of 2 fm.

Hints: using units $\hbar = c = 1$,

$$1\ \text{fm}^{-1} \approx 197\ \text{MeV}$$
$$\alpha = e^2/\hbar c \approx 1/137$$

Question 2 (15 marks)

(a) Using the attached picture of single particle energy levels, find the shell model configuration for the ground state of $^{26}_{12}$Mg and $^{27}_{13}$Al. What are the parities and spins of these nuclei?

(b) Calculate the magnetic moment of these nuclei using the formula

$$\mu = \begin{cases} \mu_N\left[g_l(j-1/2) + \frac{1}{2}g_s\right], & j = l + \frac{1}{2} \\ \mu_N\left[g_l\frac{j(j+3/2)}{j+1} - \frac{1}{2}\frac{j}{j+1}g_s\right], & j = l - \frac{1}{2} \end{cases}$$

where $g_l = 1$, $g_s = 5.6$ for a proton and $g_l = 0$, $g_s = -3.8$ for a neutron.

(c) The first two excited states of $^{27}_{13}$Al have quantum numbers $J^P = 1/2^+$ and $3/2^+$. (The energies of these states are 0.844 MeV and 1.014 MeV, respectively.) Find the shell model configuration for these states.

(d) Calculate the expectation value of the spin-orbit contribution to the Hamiltonian $H_{ls} = a\,(\boldsymbol{l} \cdot \boldsymbol{s})$ in the ground and second-excited states, and hence find the value of the spin-orbit constant a for this spin-orbit pair.