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THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS3050 – Nuclear Physics

Session 2, 2010

1. Time allowed – 2 hours
2. Total number of questions – 4
3. Total marks available – 60
4. Answer ALL questions
5. ALL QUESTIONS ARE OF EQUAL VALUE.
   Marks available for each question are shown in the examination paper.
6. University-approved calculators may be used.
7. All answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
8. This paper may be retained by the candidate.

PHYS3050 — Useful Formulae and Tables

Table of quark properties:

Quark type (flavour)	u	d	s	c
Baryon number B	1/3	1/3	1/3	1/3
Spin J	1/2	1/2	1/2	1/2
Charge Q (units of e)	+2/3	−1/3	−1/3	+2/3
Isospin T	1/2	1/2	0	0
Isospin projection T_z	+1/2	−1/2	0	0
Strangeness S	0	0	−1	0
Charm C	0	0	0	+1

Some useful formulae:

- Radial Schrödinger equation for a central potential, letting $\psi(r,\theta,\phi) = \frac{R_l(r)}{r} Y_{lm}(\theta,\phi)$:

$$\frac{d^2 R_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right) R_l(r) = 0 \ .$$

- Density of states formula:

$$dn = \frac{4\pi p^2}{(2\pi\hbar)^3}\, dp$$

- $E^2 = m^2c^4 + p^2c^2$

- Wavefunction of K-shell electron ($1s$ electron):

$$\psi(r) = \sqrt{\frac{Z^3}{\pi a_B^3}}\exp(-Zr/a_B), \qquad a_B = \frac{\hbar^2}{m_e e^2}$$

Particle properties:

	Q	J^P	B	T	S	C
p	1	$1/2^+$	1	$1/2$	0	0
n	0	$1/2^+$	1	$1/2$	0	0
π^+	1	0^-	0	1	0	0
π^o	0	0^-	0	1	0	0
π^-	-1	0^-	0	1	0	0
K^+	1	0^-	0	$1/2$	1	0
K^-	-1	0^-	0	$1/2$	-1	0
K^o	0	0^-	0	$1/2$	1	0
K^o_S	0	0^-	0	$1/2$		0
K^o_L	0	0^-	0	$1/2$		0
η	0	0^-	0	0	0	0
ρ^+	1	1^-	0	1	0	0
ρ^o	0	1^-	0	1	0	0
ρ^-	-1	1^-	0	1	0	0
ω	0	1^-	0	0	0	0
Λ^o	0	$1/2^+$	1	0	-1	0
Σ^-	-1	$1/2^+$	1	1	-1	0
Σ^o	0	$1/2^+$	1	1	-1	0
Σ^+	1	$1/2^+$	1	1	-1	0
Δ^-	-1	$3/2^+$	1	$3/2$	0	0
Δ^o	0	$3/2^+$	1	$3/2$	0	0
Δ^+	1	$3/2^+$	1	$3/2$	0	0
Δ^{++}	2	$3/2^+$	1	$3/2$	0	0
Ξ^o	0	$1/2^+$	1	$1/2$	-2	0
Ξ^-	-1	$1/2^+$	1	$1/2$	-2	0
Ω^-	-1	$3/2^+$	1	0	-3	0
J/ψ	0	1^-	0	0	0	0
D^+	1	0^-	0	$1/2$	0	1
D^-	-1	0^-	0	$1/2$	0	-1
D^o	0	0^-	0	$1/2$	0	1

Question 1 (15 marks)

(a) Estimate the depth of the proton-neutron potential approximated by the spherical square well. It is known from experiment that the binding energy is close to zero (deuteron), nucleon mass $m \approx 940$ MeV, and the radius of the potential $R \approx 2$ fm.

(b) Compare your answer for (a) with the Coulomb interaction of two protons at a distance of 2 fm.

Hints: using units $\hbar = c = 1$,

$$1\ \text{fm}^{-1} \approx 197\ \text{MeV}$$
$$\alpha = e^2/\hbar c \approx 1/137$$

Question 2 (15 marks)

(a) Starting from the operator of the magnetic moment $\boldsymbol{\mu} = \mu_N(g_l \boldsymbol{l} + g_s \boldsymbol{s})$, derive the shell model formulae for the magnetic moment of an even-odd nucleus

$$\mu = \begin{cases} \mu_N \left[g_l \left(j - \frac{1}{2} \right) + \frac{1}{2} g_s \right] & \text{if } j = l + \frac{1}{2} \\ \mu_N \left[g_l \frac{j(j+\frac{3}{2})}{j+1} - \frac{j}{2(j+1)} g_s \right] & \text{if } j = l - \frac{1}{2} \end{cases}$$

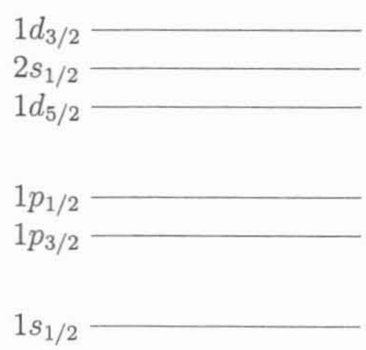
where $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$ is the angular momentum of the unpaired nucleon. For protons $g_l = 1$, $g_s = 5.6$ and for neutrons $g_l = 0$, $g_s = -3.8$.

(b) Use the shell model energy levels given below to compute

i. the ground state angular momentum, isospin projection, parity, and magnetic moment of the ${}^{11}_{6}$C, ${}^{12}_{6}$C, and ${}^{13}_{6}$C nuclei.

ii. the angular momentum, isospin projection, and parity for the excited ${}^{11}_{6}$C and ${}^{13}_{6}$C nuclei with an unpaired nucleon in the first excited state.

iii. Is it possible for an E1 transition to take place between these excited states and the corresponding ground states? Explain.



Question 3 (15 marks)

(a) For the following β-decays state whether the decay will be Fermi type, Gamow-Teller type, both, or forbidden. Give the reasons.

$$
\begin{aligned}
{}^{19}_{10}\mathrm{Ne}\ (1/2^+) &\rightarrow {}^{19}_{9}\mathrm{F}\ (1/2^+) \\
{}^{20}_{9}\mathrm{F}\ (2^+) &\rightarrow {}^{20}_{10}\mathrm{Ne}\ (0^+) \\
{}^{152}_{63}\mathrm{Eu}\ (3^-) &\rightarrow {}^{152}_{62}\mathrm{Sm}^*\ (2^+) \\
{}^{14}_{8}\mathrm{O}\ (0^+) &\rightarrow {}^{14}_{7}\mathrm{N}^*\ (0^+) \\
{}^{6}\mathrm{He}\ (0^+) &\rightarrow {}^{6}\mathrm{Li}\ (1^+) \\
n &\rightarrow p \\
{}^{3}\mathrm{H}\ (1/2^+) &\rightarrow {}^{3}\mathrm{He}\ (1/2^+)
\end{aligned}
$$

(b) Using the Fermi golden rule

$$\lambda = 2\pi \left|V_{if}\right|^2 \rho_f, \qquad \rho_f = \frac{dn}{dE}$$

find the dependence of the β-decay probabilities on electron energy E when $E \ll m_e$ (the electron is non-relativistic).

(c) Give evidence that the weak interaction does not conserve parity.

Question 4 (15 marks)

Use the table of particle properties provided to

(a) Give the quark assignments for the following particles:

$$\mathrm{p},\ \mathrm{n},\ \pi^+,\ \eta,\ K^+,\ \Sigma^-,\ \Delta^-,\ \Omega^-,\ \mathrm{J}/\psi,\ \mathrm{D}^+.$$

Using Ω^- as an example, explain why the extra quantum number, colour, is necessary.

(b) State whether the following reactions proceed via the strong, weak, or electromagnetic interactions, or are forbidden. Present corresponding Feynman diagrams for these reactions at the quark-lepton level (with intermediate particles g, γ, W, Z).

$$
\begin{aligned}
\bar{\nu}_e + p &\rightarrow n + e^+ \\
\rho^+ &\rightarrow \pi^+ + \pi^o \\
\pi^- &\rightarrow e^- + \gamma \\
\pi^o &\rightarrow 2\gamma \\
\Lambda^o &\rightarrow \pi^- + p \\
\pi^+ &\rightarrow \mu^+ + \nu_\mu \\
\mathrm{J}/\psi &\rightarrow \mu^+ + \mu^-
\end{aligned}
$$

(c) Determine the CP-parity of the $\pi^o\pi^o$ pair with orbital angular momentum $l = 0$. Explain why experiments with K-meson decays demonstrated CP-violation.