Mid session test 2006
Quantum Mechanics PHYS3210
Time: 1 hour.
Total number of questions 2. Answer both questions.
This paper may be retained by the candidate.

Question 1

Consider bound states of a particle mass m in a square well potential

$$V(x) = \begin{cases} 0, & x < -a/2 \\ -V_0, & -a/2 < x < a/2 \\ 0, & x > a/2 \end{cases} ,$$

a) (1marks) Write down wave function of the particle in general form at $|x| < a/2$ and at $|x| > a/2$.
b) (2marks) Explain in few words why energy of a bound state is always negative.
c) (2marks) Explain why the wave function of a bound state in this potential must have definite parity.
d) (3marks) It is known that the wave function and the first derivative of the wave function must be continuous. Apply these conditions to the present problem and hence derive algebraic equations that describe energies of bound states. Do it separately for even (positive parity) states and for odd (negative parity) states.
e) (2marks) Using algebraic equation derived in the part **d** for even states demonstrate graphically that there is always at least one bound state in the well. *Do not solve the equation, just demonstrate existence of one or more bound states.*

Question 2

Consider a particle bound in a double well potential. The wells are identical and separated by a large distance. Therefore the wave functions of stationary states are given by symmetric and antisymmetric wave functions

$$\psi_+ \approx \frac{1}{\sqrt{2}}(\psi_1 + \psi_2) , \quad \psi_- \approx \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) ,$$

where ψ_1 and ψ_2 are states localized in wells. The small energy splitting between the stationary states, $\delta\epsilon = \epsilon_- - \epsilon_+$ it is due to tunneling between the wells. The energy splitting is known. At $t = 0$ the system is prepared in the quantum state with wave function

$$\psi(t = 0) = \sqrt{\frac{2}{3}}\psi_1 + \sqrt{\frac{1}{3}}\psi_2 .$$

So initially the probability to locate particle in the first well is 2/3 and the probability to locate particle in the second well is 1/3.
a) (5marks) Find wave function of the system $\psi(t)$ at $t > 0$.
b) (5marks) Find probabilities $P_1(t)$ and $P_2(t)$ to locate particle in the first well and in the second well at time t. The answer depends only on time and on the energy splitting $\delta\epsilon$.