Mid session test 2005
Quantum Mechanics PHYS3010/PHYS3210
Time: 1 hour.
Total number of questions 2. Answer all questions.
The questions are of equal value.
Calculators may be used.
This paper may be retained by the candidate.

Question 1

a) (5marks) Consider $V(x) = g\delta(x)$, where $g = -|g|$ is negative. Find the wave function and the energy of the bound state, do not forget to normalize the wave function. You can use without proof the matching conditions at the δ-function potential.

$$\psi_+ = \psi_- \ , \quad \psi'_+ - \psi'_- = \frac{2mg}{\hbar^2}\psi \ .$$

b) (2marks) Consider a particle with positive energy ($\epsilon > 0$) moving in 1D potential $V(x)$. The potential vanishes at $|x| > a$ where a is some characteristic length.
Formulate the scattering problem and write down the wave function of the particle at $x < -a$ and $x > a$.
c) (1mark) Clearly indicate which part of the wave function corresponds to the incident wave, reflected wave, and transmitted wave.
d) (1mark) Define the reflection amplitude R and transmission amplitude T. How are the amplitudes related to transmission and reflection probabilities?
e) (1mark) What can you say about $|R|^2 + |T|^2$? Justify your answer.

Question 2

A particle is moving in a harmonic potential, $\hat{H} = \frac{p^2}{2\mu} + \frac{m\omega^2 x^2}{2}$. The energies and the wave functions of the ground state and of the first excited state are

$$\epsilon_0 = \frac{\hbar\omega}{2}, \quad \psi_0 = (\alpha/\pi)^{1/4} e^{-\alpha x^2/2},$$
$$\epsilon_1 = \frac{3\hbar\omega}{2}, \quad \psi_1 = (\alpha/\pi)^{1/4} \sqrt{2\alpha}\ x\ e^{-\alpha x^2/2},$$

where $\alpha = \frac{m\omega}{\hbar}$. Initially, at $t = 0$, the sytem is prepared in the quantum state

$$\psi(x, t = 0) = \sqrt{\frac{2}{3}}\psi_0 + \sqrt{\frac{1}{3}}\psi_1 \ .$$

a) (2marks) Calculate $\psi(x, t)$
b) (1mark) Calculate the average energy of the particle.
c) (3 marks) Calculate the average position of the particle $\langle\hat{x}\rangle$ as a function ot time. Here $\langle ...\rangle = \langle\psi(x, t)|...|\psi(x, t)\rangle$.
d) (4 marks) Calculate the average momentum of the particle $\langle\hat{p}\rangle$ as a function ot time.

You can use without proof the following integrals

$$\int_{-\infty}^{+\infty} e^{-y^2} dy = \sqrt{\pi}, \quad \int_{-\infty}^{+\infty} y^2 e^{-y^2} dy = \frac{\sqrt{\pi}}{2}.$$