THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

UNSW
>014407892

**PHYS2060 THERMAL PHYSICS**

**FINAL EXAMINATION**
**SESSION 2 - NOVEMBER 2010**

1. Time allowed = 2 hours

2. Total number of questions = 5

3. Total number of marks =65

4. All questions are NOT of equal value. Marks available for each question are shown in the examination paper.

5. Answer all questions.

6. All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

7. The use of UNSW-approved calculators is allowed.

8. This paper may be retained by the candidate.

**The following information is supplied as an aid to memory.**

Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number $N_A = 6.022 \times 10^{23}$ mol$^{-1}$
Real gas constant $R = 8.314$ J/K.mol

Specific heat of liquid $H_2O$ = 4.18 J/gK
Latent heat of the liquid-solid transition for $H_2O$ = 333 J/g
Adiabatic constant for $N_2$ $\gamma = 1.4$
Molar mass of air = 29 g/mol

Ideal gas equation $PV = nRT = Nk_BT$

Maxwell's velocity distribution

$$D(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)$$

Thermodynamic Potentials

$$F = U - TS \qquad dF = dU - TdS - SdT$$
$$G = U - TS + PV \qquad dG = dU - TdS - SdT + PdV + VdP$$
$$dU = TdS - PdV$$

Clausius-Clapeyron Equation $\frac{dP}{dT} = \frac{L}{T\Delta V}$

**Question 1 [ 14 Marks]**

(a) State the zeroth law of thermodynamics and explain how it follows from the zeroth law that temperature is a universal property: one that is possessed in common by all systems whatever their nature.

(b) During a hailstorm, hailstones with an average mass of 3g and a speed of 14 m/s strike a windowpane at an angle of 45° C. The area of the window is 0.4 $m^2$. And the hailstones hit at a rate of 25 per second. What average pressure do they exert on the window? How does this compare to the pressure of the atmosphere?

(c) From equipartition of energy we know that:

$$\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} k_B T$$

Using this, derive an expression for the root-mean-square speed of a gas molecule.

(d) Uranium has two common isotopes, with atomic masses of 238 and 235. One way to separate these isotopes is to combine the uranium with fluorine to make uranium hexafluoride ($UF_6$) gas, and then exploit the difference in the average thermal speeds of molecules containing the different isotopes. Calculate the rms speed of each type of molecule (i.e. both for $^{235}UF_6$ and $^{238}UF_6$ ) at a temperature of 20° C, and state the expected $v_{rms}$ difference. Assume the atomic weight of F is 19 u (atomic mass units). Remember that 1 u = 1.66 x $10^{-27}$ kg.

(e) List all the degrees of freedom (or as many as you can) for a diatomic molecule such as CO in the gaseous phase. How many in total do you expect? How much energy to you expect to be in each mode, and hence what will the total internal energy of the molecule in terms of $k_B$ and T?

(f) Sketch a graph of specific heat v. temperature for the diatomic molecule in the gaseous phase. The y-axis should be in multiples of $k_B$, but you will not be able to put quantitative units on the x-axis. Mark on the graph where the different degrees of freedom start to contribute (or stop contributing) to C. (Hint: remember that as temperature increases, more degrees of freedom tend to be excited.)

**Question 2 [14 Marks]**

(a) Give a statement of the first law of thermodynamics. What is the basic underlying principal of this law?

(b) Briefly explain the following statement, referring where necessary to the system and its surroundings: "The internal energy is a state variable of a system but heat and work are transfer variables instead."

(c) Given that

$$W = -\int P dv$$

Derive an expression for the work done in an isothermal process, as a function of the number of moles of a gas and the temperature of the gas.

(d) Calculate the work done when 0.1 mol of an ideal gas at 300K expands isothermally from a volume of $2 \times 10^{-3}$ m$^3$ to $10 \times 10^{-3}$ m$^3$. Is work done on or by the gas during this process?

(e) In the course of pumping up a car tire, 10 litres of air at atmospheric pressure is compressed adiabatically to a pressure of 6 atm (air has $\gamma = 1.4$)

(i) Plot the process of compressing the air in the pump on a $PV$-diagram. Be sure to include the direction of the process and how it relates to the isotherms corresponding to the initial and final temperatures of the process.

(ii) What is the final volume of the air after the compression?

(iii) If the temperature of the air is initially 290 K, what is the temperature after compression?

**Question 3 [14 Marks]**

A heat engine is any device that absorbs heat and converts part of that energy into useful work.

(a) Draw an energy flow diagram for a heat engine showing the hot reservoir, the cold reservoir and the various energy flows involved. Define the efficiency of the heat engine.

(b) Using the first law, show that the efficiency $e$ can be written $e = 1 - Q_c/Q_h$, where $Q_h$ and $Q_c$ are the energy flows from the hot reservoir to the engine and from the engine to the cold reservoir respectively.

(c) A car-like engine operates in a Carnot cycle. The engine starts its cycle at point A, where the cylinder has a maximum volume of $V_A = 0.7$ L, and an initial pressure $P_A = 100$ kPa, temperature $T_A = 290$ K. The cylinder contains pure nitrogen with $\gamma = 1.4$ and a molar mass of 28 g/mol:

(i) In one or two sentences, briefly explain the significance of the Carnot cycle in thermodynamics.

(ii) Calculate the mass of nitrogen in the cylinder at A. Assume this mass will remain constant for the rest of the question.

(iii) Sketch a diagram of the Carnot cycle. Use the point A as the maximum volume (parameters given above) then label the start of the next legs (processes) of the cycle B, C, D. Draw isotherms clearly on you diagram, and show the relationship of the cycle to these. Label the type of expansion or contraction (isothermal / adiabatic) on each process/leg of the cycle.

(iv) Give a brief description of each process, referring to your diagram.

(v) For the process A → B, $V_B = 1/5\ V_A$. Hence calculate $P_B$ and $T_B$.

(vi) For the process B → C, $V_C = 1/2\ V_B$. Hence calculate $P_C$ and $T_C$.

**Question 4 [14 Marks]**

(a) When the sun is high in the sky, it delivers approximately 1000 watts of power to each square metre of the Earth's surface. The temperature of the surface of the Sun is about 6000K and the surface of the Earth about 300K. Estimate the entropy created in one year by the flow of solar heat onto a square metre of the Earth. Assume that an average of 8 hours sunlight per day.

(b) A power plant that produces 1GW ($10^9$ watts) of electricity, the steam turbines take in steam at a temperature of 500°C, and the waste heat is expelled into the environment at 20°C.

   (i) What is the maximum possible efficiency of the plant?

   (ii) Suppose you develop a new material for making pipes and turbines which allows the maximum steam temperature to be raised to 600°C. By what percentage does this improve the efficiency of the plant, assuming no extra fuel is consumed to produce the higher temperature.

(c) A cylinder contains one litre of air at room temperature and atmospheric pressure. At one end of the cylinder is a massless piston, whose surface area is 0.01 $m^2$. The piston is pushed in very suddenly, exerting a force of 2000N. The piston moves only one millimetre, before it is stopped by an immovable barrier of some sort.

   (i) How much work have you done on this system?

   (ii) How much heat has been added to the gas?

   (iii) Assuming that all the energy added goes into the gas (not the piston or cylinder walls), by how much does the internal energy of the gas increase?

   (iv) Use $dU = TdS - PdV$ to calculate the change in the entropy of the gas (once it has reached equilibrium).

**Question 5 [9 Marks]**

(a) Using the definition of specific heat, the first law of thermodynamics and the ideal gas law, show that:

  (i) $dQ = C_v dT + PdV$, where $C_V$ is the specific heat at constant volume,

  (ii) $C_P = C_V + nR$, where $C_P$ is the specific heat at constant pressure and $R$ is the ideal gas constant.

(b) At a phase boundary, a material is equally stable in either of the two phases and hence the Gibbs free energy $G = U + PV - TS$ is equal in the two phases. Starting from this fact, derive the Clausius-Clapeyron relation:

$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$

where $L$ is the latent heat of the transition and $\Delta V$ is the change in volume during the transition.

(c) The density of water at 1000 kg/m$^3$, is slightly higher than that of ice at 917 kg/m$^3$. Use the Clausius-Clapeyron relation to explain why the slope of the phase boundary between water and ice is negative.