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THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – NOVEMBER 2009

PHYS2060 – THERMAL PHYSICS

Time allowed – 2 hours

Total number of questions – 4

Attempt **ALL** questions

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}
Real gas constant $R = 8.314$ J/K.mol

Specific heat of liquid H_2O = 4.18 J/gK
Latent heat of the liquid-solid transition for H_2O = 333 J/g
Latent heat of the liquid-gas transition for H_2O = 2270 J/g
Adiabatic constant for N_2 g = 1.4
Specific heat at constant pressure for N_2 C_P = 29.12 J mol^{-1} K^{-1}
Specific heat at constant pressure for N_2 C_V = 20.8 J mol^{-1} K^{-1}
Molar mass of air = 29 g/mol

Partial derivatives

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \; ; \quad \left(\frac{\partial y}{\partial x}\right)_z = \frac{1}{\left(\frac{\partial x}{\partial y}\right)_z} \; ; \left(\frac{\partial y}{\partial x}\right)_z = -\frac{\left(\frac{\partial z}{\partial x}\right)_y}{\left(\frac{\partial z}{\partial y}\right)_x}$$

Thermodynamic quantities

$\kappa_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ Isothermal compressibility $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ Expansivity

$\kappa_S = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_S$ Adiabatic compressibility $c_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V$

$$c_P = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_P + \frac{P}{n}\left(\frac{\partial V}{\partial T}\right)_P$$

Thermodynamic potentials

$F=U-TS$	$dF=-SdT-PdV$	Helmholtz Free Energy
$H=U+PV$	$dH=TdS+VdP$	Enthalpy
$G=U-TS+PV$	$dG=-SdT+VdP$	Gibbs Free Energy

Ideal gas

$PV=NkT$

$$U = \frac{1}{\gamma - 1} NkT$$

$PV^{\gamma} = const$ for Adiabatic process

Efficiency of Carnot cycle

$$\eta = \frac{W_{cycle}}{Q_{in}} = 1 - \frac{T_c}{T_h}$$

QUESTION 1

The van der Waals equation of state for a fluid is given by:

$$\left(P+\frac{a}{v^2}\right)(v-b)=RT$$

where a,b are characteristic constants for a given substance.

(i) Sketch a P-v diagram for a van der Waals fluid showing a representative set of isotherms.

(ii) On your sketch, clearly indicate which regions correspond to: gas, liquid and a mixed state (gas + liquid).

(iii) Mark the critical point on your P-v diagram.

(iv) Explain how the equations:

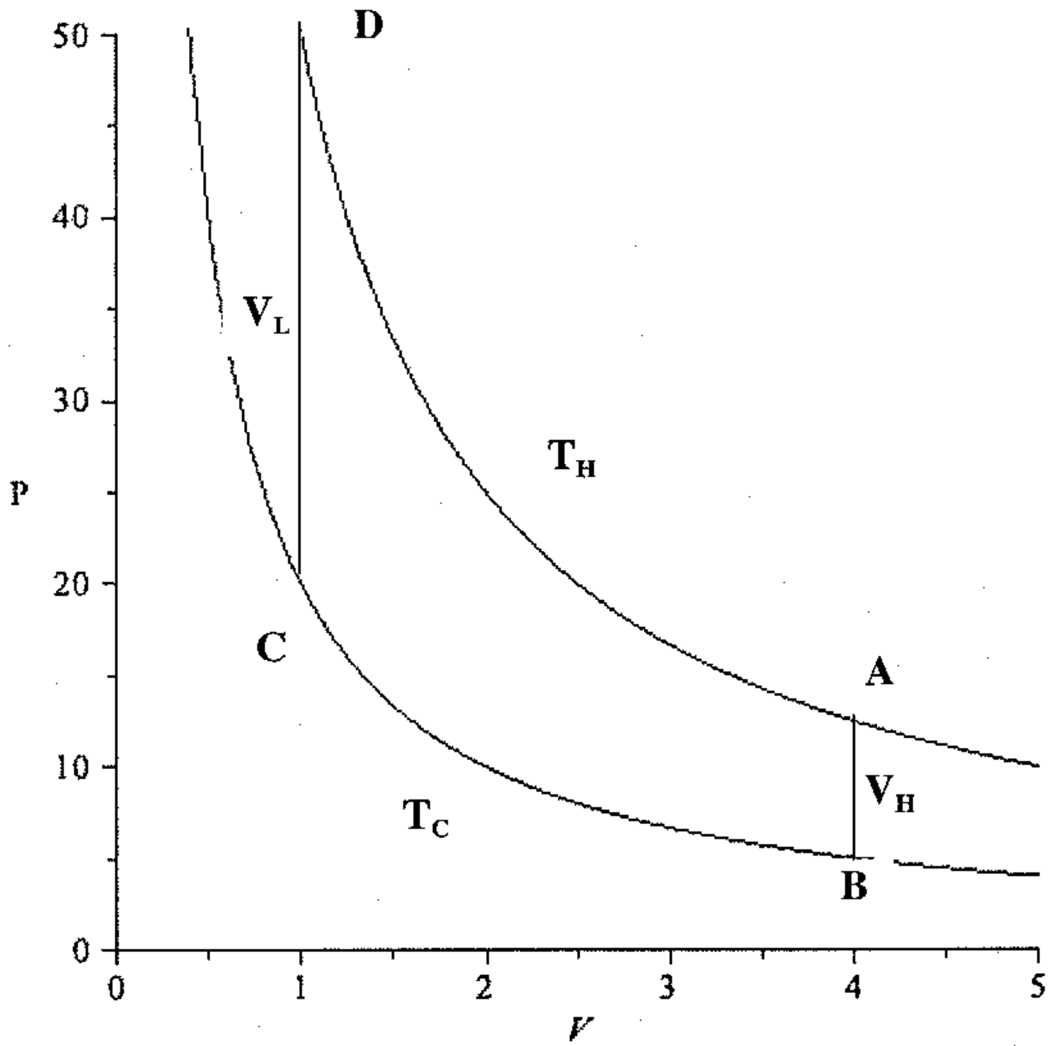
$$\left(\frac{\partial P}{\partial v}\right)_T=0$$

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_T=0$$

are related to the critical point.

(v) Using these equations, derive expressions for the critical specific volume, temperature and pressure (v_C, T_C and P_C) in terms of the parameters a,b and R.

QUESTION 2



A Stirling heat engine works on the following cycle: A→B – an isochoric (constant volume) depressurisation at volume, V_H; B→C – an isothermal compression at a low temperature T_C; B→C – an isochoric pressurisation at volume V_L; and an isothermal expansion at T_H. This engine is effectively working between two heat reservoirs, one at T_H and one at T_C. Assume that the engine is filled with **n** moles of an ideal gas, where the adiabatic gas constant is γ.

(i) Derive an expression for the work done by the engine in one complete cycle in terms of the engine parameters: V_H, V_L, T_H, T_C, **n** and γ.

(ii) Using the First Law of Thermodynamics (or otherwise), derive an expression for the heat input to the engine during the isothermal expansion step D→A in terms of the engine parameters: V_H, V_L, T_H, T_C, **n** and γ.

(iii) Using the First Law of Thermodynamics (or otherwise), calculate the heat input into the engine during the isochoric (constant volume) pressurisation step C→D in terms of the engine parameters: V_H, V_L, T_H, T_C, **n** and γ.

(iv) Using your results from (i)-(iii), derive an expression for the efficiency of the Stirling engine as a function of only three parameters: γ plus the ratio of temperatures $\frac{T_C}{T_H}$ and the ratio of volumes $\frac{V_H}{V_L}$ (called the compression ratio).

(v) Using this expression, compare the efficiency of the Stirling engine with the Carnot cycle.

QUESTION 3

PART A

(i) Explain the meaning of the terms: extensive parameters and intensive parameters. Give examples.

(ii) What is meant by a quasi-static process? Give an example.

(iii) Heat flux and work are represented by inexact or improper differentials. What does this mean? How do these parameters differ from the state parameters?

PART B

In 1824, a French engineer named Sadi Carnot described a theoretical engine, now called the Carnot engine, which is of great importance from both theoretical and practical viewpoints.

(iv) Draw the P-V diagram for the Carnot cycle indicating key features of the cycle. In your diagram, indicate parts of the cycle where heat is added and removed. Indicate the direction of the cycle such that it produces net work.

For the Carnot cycle, a quantity of heat Q_H is transferred from a high temperature heat bath at temperature T_H, while a quantity of heat Q_C is lost to a low temperature heat bath at temperature T_C. The analysis of the Carnot cycle shows that:

$$\left|\frac{Q_H}{T_H}\right| = \left|\frac{Q_L}{T_L}\right|$$

(v) What is the meaning of this equation?

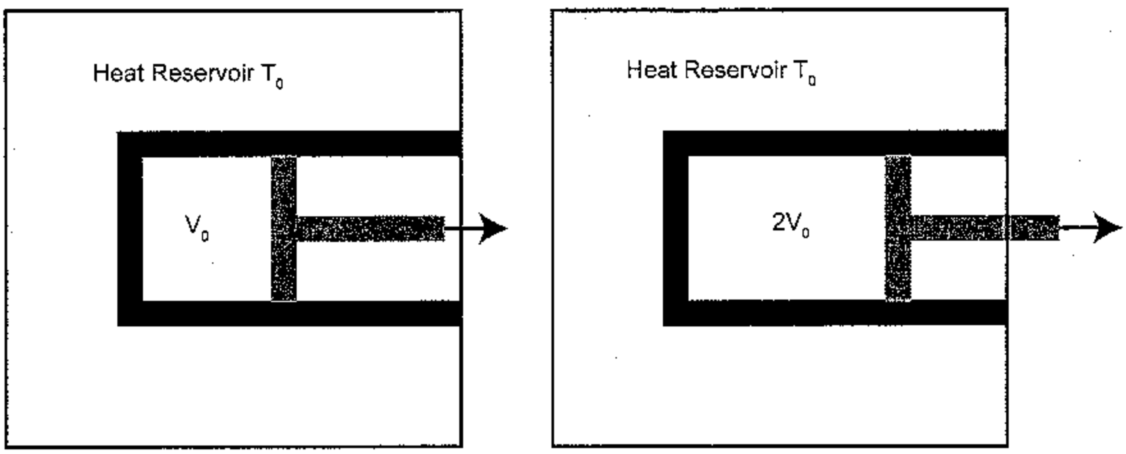
(vi) What is this parameter $\frac{Q}{T}$? What is it called?

(vii) Under what conditions is this parameter conserved?

QUESTION 4

PART A

An ideal monatomic gas is contained in a piston whose state is characterised by the extensive parameters (U_0, S_0, V_0, N_0) and intensive parameters (T_0, P_0, μ_0). The system is placed in thermal contact with a heat reservoir at constant temperature T_0, hence, all processes are isothermal. The plunger in the piston moves slowly out of the piston so that the gas expands quasi-statically to a final volume which is twice the initial volume, $V_f = 2V_0$.



(i) Calculate the work done by the gas as it expands from V_0 to $V_f = 2V_0$.

(ii) Prove that the internal energy U does not change during this expansion.

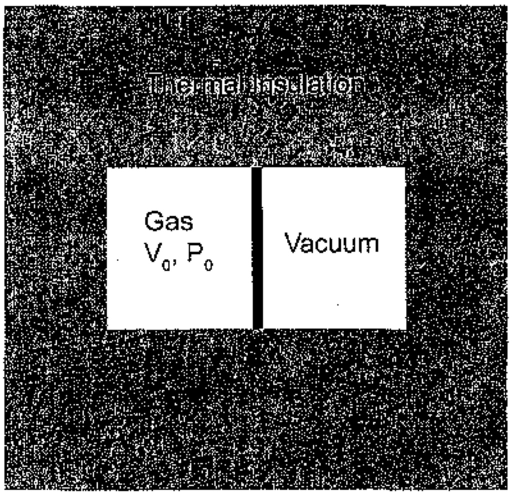
(iii) Calculate the amount of heat into the gas during this isothermal expansion.

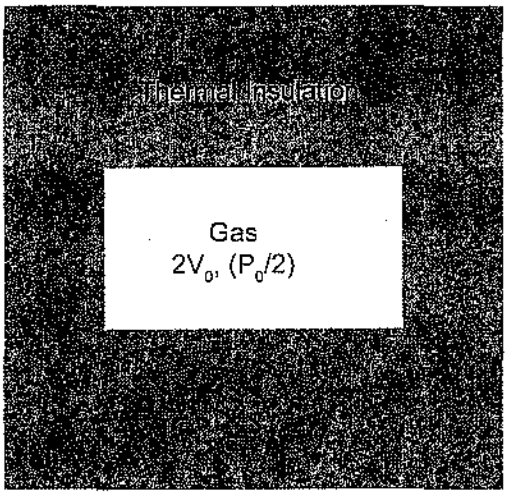
(iv) Calculate the change in entropy that occurs during the expansion.

(v) What is the source of this entropy increase?

PART B

A new cylinder is wrapped in thermal insulation so that all processes are adiabatic. This cylinder has two chambers separated by a thin membrane. On the left side of the partition is an ideal monatomic gas that is in an identical state to the one described at the start of Part A. It is described by the same state parameter values: (U_0, S_0, V_0, N_0) and (T_0, P_0, μ_0). The chamber on the right side of the partition has the same volume V_0, but it is evacuated (empty). The partition separating the chambers is broken, so that the gas expands irreversibly to fill the whole chamber of volume $V_f = 2V_0$.





(vi) Determine the change of entropy that occurs during this expansion.

(vii) How does it compare with the quasi-static isothermal case in Part A.

(viii) What is the source of entropy increase?