P-S

THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

NOVEMBER 2005

PHYS2060

Thermal Physics

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions are of equal value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the

Enrolment and Assessment Section: Calculators

Answers must be written in ink. Except where they

are expressly required, pencils may only be used

for drawing, sketching or graphical work

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT \qquad PV^{\gamma} = \text{constant}$$

$$B \equiv -V\left(\frac{\partial P}{\partial V}\right) = \frac{1}{\kappa} \qquad \alpha \equiv \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_t \qquad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \qquad \gamma \equiv \frac{c_P}{c_V} = \frac{f+2}{f}$$

$$b = \frac{2}{3}N_A\pi d^3 \qquad l = \frac{1}{n\sigma}$$

$$\Delta N_v = \frac{4N}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)\Delta v$$

$$H \equiv U + PV \qquad F \equiv U - TS \qquad G \equiv F + PV$$

$$dU = TdS - PdV, \quad dH = TdS + VdP, \quad dF = -SdT - PdV, \quad dG = -SdT + VdP$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \qquad \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_P = \left(\frac{\partial P}{\partial T}\right)_S \qquad \left(\frac{\partial S}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_S$$

$$\left(\frac{\partial P}{\partial T}\right)_{23} = \frac{L_{23}}{T(v_3 - v_2)} \qquad \left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial V}\right)_P\left(\frac{\partial V}{\partial P}\right)_T = -1$$

$$TdS = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV \qquad TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \qquad \int_0^\infty e^{-ax}\,dx = \frac{1}{a}$$

$R = 8.31\ 10^3$ J kilomole^{-1} K^{-1} $\qquad k_B = 1.38\ 10^{-23}$ JK^{-1}

$N_A = 6.02\ 10^{26}$ molecules kilomole^{-1}

1 Atmosphere $\cong$ 101 kPa

1 kg mole of an ideal gas occupies 22.4 m^3 at 273 K and 100 Pa.

Question 1

Consider a model of an ideal gas adiabatic atmosphere. This means that there is no heat transfer in the atmosphere and hence pressure and temperature vary with altitude h but the combination $p^{1-\gamma}T^{\gamma}$ remains *constant*. This is a reasonable model for the Earth's atmosphere below an altitude about 10-15 km in the condition of rapid convection.

a. **(4 marks)** Using the equation of mechanical equilibrium together with $p^{1-\gamma}T^{\gamma} = const$ and together with the equation of state of the ideal gas, derive differential equations which describe the variation of temperature and pressure with altitude.

b. **(4 marks)** Solve the equations derived in part **(a)** and hence find $p(h)$ and $T(h)$.

c. **(2 marks)** Pressure and temperature at sea level are $p(h = 0) = p_0 = 1atm \approx 10^5 N/m^2$, $T(h = 0) = T_0 = 27C = 300K$. Find the values of temperature and pressure at $h = 10km$.

The molar mass of air is $\mu \approx 29$, the adiabatic parameter is $\gamma \approx 7/5$, and the universal gas constant is $R = 8.31\ 10^3 J/kilomole/K$.

Question 2

A nucleus with spin 3/2 has four different quantum states corresponding to different orientations of the spin. The nucleus is put in an external electric field (for example the electric field of some crystal lattice). As a result, the energy of the first two quantum states is $\epsilon_1 = \epsilon_2 = 0$, the energy of the third and the fourth states is $\epsilon_3 = \epsilon_4 = \epsilon$. Here $\epsilon/k = 1mK$, where mK is milli-Kelvin, and $k = 1.3810^{-23} J/K$ is the Boltzmann constant. Consider an ensemble of nuclei in a heat bath with temperature T.

a. **(4 marks)** Derive expressions for the probabilities to find the nucleus in each particular quantum state.

b. **(3 marks)** Calculate the average energy E of the nucleus.

c. **(3 marks)** Sketch the plot of the specific heat capacity c per nucleus versus temperature and calculate the value of c at $T = 0.2mK$.

Question 3

a. **(5 marks)** Using Maxwell's relations or otherwise, prove that

$$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV,$$

$$TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP,$$

where C_V and C_P are the heat capacities and where all other terms have their usual meanings.
b. **(5 marks)** Consider one cubic cm of metallic cooper at room temperature, $T = 300K$. The density of Cu is $\rho = 8.96 \times 10^3 kg\ m^{-3}$, its thermal coefficient of volume expansion is $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = 50.1 \times 10^{-6}\ K^{-1}$, and its specific heat is $c_p = 384J\ kg^{-1}\ K^{-1}$. Suddenly a hydrostatic pressure $p = 10^3 atm \approx 10^8 N/m^2$ is applied to the cube. Find by how much does the temperature of the cube rise?
Hint: A fast process can be considered as an adiabatic one and part **(a)** could be helpful.

Question 4

a. **(2 marks)** State the condition of phase equilibrium in terms of chemical potentials.
b. **(4 marks)** Starting from the 1st law of thermodynamics in the form $d\mu = -sdT + vdP$ derive the Clausius Clapeyron equation

$$\frac{dP}{dT} = \frac{l_{ab}}{T(v_b - v_a)}.$$

(Hint: the answer to part **a** is helpful) Explain the meaning of this equation.
c. **(4 marks)** Consider the water - vapor transition assuming the vapor be an ideal gas. Assume also that the heat of transformation is T-independent. Starting from the Clausius Clapeyron equation, derive an expression for the vapor pressure as a function of the transition temperature. You may assume $v_{liquid} \ll v_{gas}$ or $\rho_{liquid} \gg \rho_{gas}$.