P. 5

THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

NOVEMBER 2004

PHYS2060

Thermal Physics

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions are of equal value

This paper may be retained by the candidate

Candidates may not bring their own calculators

The following materials will be provided by the Enrolment and Assessment Section: Calculators

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT \qquad PV^{\gamma} = \text{constant}$$

$$B \equiv -V\left(\frac{\partial P}{\partial V}\right) = \frac{1}{\kappa} \qquad \alpha \equiv \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_{\tau} \qquad \beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \qquad \gamma \equiv \frac{c_P}{c_V} = \frac{f + 2}{f}$$

$$b = \frac{2}{3} N_A \pi d^3 \qquad l = \frac{1}{n\sigma} \qquad \lambda_T \equiv \frac{h}{\sqrt{2\pi m k T}}$$

$$\frac{\bar{N}_j/N}{g_j} = \exp\frac{\mu - \varepsilon_j}{k_B T} \qquad \Delta N_v = \frac{4N}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)\Delta v$$

$$H \equiv U + PV \qquad F \equiv U - TS \qquad G \equiv F + PV$$

$$dU = TdS - PdV, \quad dH = TdS + VdP, \quad dF = -SdT - PdV, \quad dG = -SdT + VdP$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \qquad \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_P = \left(\frac{\partial P}{\partial T}\right)_S \qquad \left(\frac{\partial S}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_S$$

$$\left(\frac{\partial P}{\partial T}\right)_{23} = \frac{L_{23}}{T(v_3 - v_2)} \qquad \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$

$$TdS = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV \qquad TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad (1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \ldots \qquad \int_0^{\infty} e^{-ax}\, dx = \frac{1}{a}$$

$R = 8.31\ 10^3$ J kilomole^{-1} K^{-1} $\qquad k_B = 1.38\ 10^{-23}$ JK^{-1}

$N_A = 6.02\ 10^{26}$ molecules kilomole^{-1}

1 Atmosphere $\cong$ 101 kPa

1 kg mole of an ideal gas occupies 22.4 m^3 at 273 K and 100 Pa.

Final test 2004
Thermal Physics 2060
Total number of questions 4. Answer all the questions.

Question 1

The van der Waals equation of state for one mole of gas is of the form

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT.$$

a. (2 marks) Using $p-v$ plot sketch a set of van der Waals isotherms. Clearly indicate regions corresponding to (i) gas, (ii) liquid and (iii) mixed state (liquid + gas).
b. (4 marks) Indicate the critical point and explain how the equations

$$\left(\frac{\partial p}{\partial v}\right)_T = 0 \ ,$$
$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0 \ ,$$

are related to the critical point
c. (4 marks) Using above equations derive expressions for the critical volume, temperature, and pressure (v_c, T_c, p_c) in terms of the parameters R, a, and b.

Question 2

A nucleus with spin 3/2 has four different quantum states corresponding to different orientations of the spin. The nucleus is put in a combination of external electric and magnetic fields. As a result the energy of the first quantum state is $\epsilon_1 = 0$, the energy of the second state is $\epsilon_2 = \epsilon$, and the energies of the third and the forth states are degenerate $\epsilon_3 = \epsilon_4 = 2\epsilon$. Here $\epsilon/k = 1mK$, where mK is milli-Kelvin ($10^{-3}K$), and k is the Boltzmann constant. Consider an ensemble of nuclei in a heat bath with temperature T.

a. (4 marks) Derive expressions for the probabilities of finding the nucleus in each particular quantum state.
b. (3 marks) Calculate the average energy E of the nucleus.
c. (3 marks) Sketch the plot of heat capacity c per nucleus versus temperature and estimate the value of c at $T = 0.2 \times mK$.

Question 3

a. (5 marks) Using Maxwell's relations or otherwise, prove that

$$TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP,$$

where C_V is the heat capacity and where all other terms have their usual meanings.

b. (5 marks) Consider a ten gram cube of metallic cooper at room temperature, $T = 300K$. The density of Cu is $\rho = 8.96 \times 10^3 kg\ m^{-3}$, its thermal coefficient of volume expansion is $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = 50.1 \times 10^{-6}\ K^{-1}$, and its specific heat is $c_p = 384 J\ kg^{-1}\ K^{-1}$. Suddenly a hydrostatic pressure $p = 10^4 atm \approx 10^9 N/m^2$ is applied to the cube. By how much does the temperature of the cube rise?

Hint: A fast process can be considered as an adiabatic one and the question **(a)** could be helpful.

Question 4

The Helmholtz free energy of a system is given by

$$F = -RTv^2 - AT^3 + \frac{B}{v},$$

where R, A, and B are some constants, v is a molar volume, and T is tempeterature.

a. (3 marks) Derive equation of state of the system.
b. (3 marks) Derive an expression for entropy of the system.
c. (2 marks) Derive an expression for the internal energy U of the system.
c. (2 marks) Derive an expression for the heat capacity at constant volume c_V.