THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS2040

Quantum Physics

JUNE 2010

1. Time Allowed: 2 hours
2. Total number of questions: 5
3. Marks available for each question are shown in the examination paper. The total number of marks is 50.
4. Attempt ALL questions!
5. Each question must be answered in a separate book clearly labeled.
6. Candidates may bring their own calculators (calculators without alphabetic keyboards).
7. Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work. Do not use red ink.
8. The paper may be retained by the candidate.

Additional information:

Planck's constant: $h = 6.626 \times 10^{-34}$ Js

Fundamental charge unit: $e = 1.60 \times 10^{-19}$ C

Vaccum speed of light: $c = 3.0 \times 10^{8}$ m/s

Mass of the Electron: $m_e = 9.1 \times 10^{-31}$ kg

Mass of the Neutron: $m_n = 1.675 \times 10^{-27}$ kg

Mass of the Proton: $m_p = 1.672 \times 10^{-27}$ kg

Boltzmann's constant: $k_B = 1.38 \times 10^{-23}$ JK^{-1}

Angstrom $\text{Å} = 1.0 \times 10^{-10}$ m

Permittivity constant: $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

Time-independent Schrodinger Equation: $-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V\Psi(x) \;=\; E\Psi(x)$

Time-dependent Schrodinger Equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V\Psi(x,t) \;=\; i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

$\int sin^2(bx)\;dx \;=\; \frac{x}{2} - \frac{sin(2bx)}{4b}$

$\int x sin^2(x)\;dx \;=\; \frac{x^2}{4} - \frac{x\;sin(2x)}{4} - \frac{cos(2x)}{8}$

$\int x^2 sin^2(bx)\;dx \;=\; \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right) sin(2bx) - \frac{xcos(2bx)}{4b^2}$

$\int\limits_{-\infty}^{\infty} e^{-bx^2} \;=\; \sqrt{\frac{\pi}{b}}$

$\sin(2\Theta) \;=\; 2\sin(\Theta)\cos(\Theta)$

Bragg's law: $2d\,\sin\Theta \;=\; n\,\lambda$

Compton Shift: $\Delta\lambda \;=\; \frac{h}{mc}\,(1-\cos\Theta)$

Energy levels in a Hydrogen-like atom with Z electrons:

$E_n \;=\; \frac{-me^4}{2\hbar^2(4\pi\varepsilon_0)^2}\,\frac{Z^2}{n^2} \;=\; -13.6\text{eV}\,\frac{Z^2}{n^2}$

Question 1 (10 marks): Electron Diffraction

(i) In order to determine the crystal structure of a solid, electrons with a wavelength of 1.2 Å can be used. Calculate the voltage of the cathode required to accelerate the electrons to this wavelength.

(ii) The distance between the atoms in a cubic crystal is for example $d = 2.3$ Å. Calculate the diffraction angles of the first, second, and third order Bragg reflection using Bragg's law: $2d \sin \Theta \; = \; n\lambda$.

(iii) Give a brief description of the experiment including a schematic figure and calculate at which distance from each other the Bragg reflections appear on a scintillator screen placed in 60 cm distance from the crystal.

(iv) How does the the distance between the first and second order Bragg reflection change when the voltage of the cathode is doubled?

Question 2 (10 marks): Infinite Quantum Well

A particle in an infinite square well with the width L has an eigenfunction of:

$$\varphi(x) \;=\; \sqrt{\frac{2}{L}}\,\sin\left(\frac{2\pi x}{L}\right) .$$

within the square well ($0 \leq x \leq L$) and zero otherwise.

(i) Determine the expectation value of x.

(ii) Calculate the probability of finding the particle within $\pm L/100$ around the center of the square well.

(iii) Calculate the probability of finding the particle within $\pm L/100$ around $x = L/4$.

(iv) Show that the particle can only emit or absorb photons whose wavelength are

$$\lambda \;=\; \frac{8mcL^2}{h}\left(\frac{1}{n'^2 - n^2}\right) .$$

where $n', n = 1, 2, 3, \ldots$ and $n' > n$.

Question 3 (10 marks): Spectral lines of Atoms

(i) Sodium ($_{11}$Na) has eleven electrons. Calculate the energy for an electronic transition from the K-shell (n = 1) to the continuum. Compare this energies with the excitation energy of an Helium-Neon Laser (633 nm). Is this energy sufficient?

(ii) What is the electronic configuration, i.e. population of the electronic levels of Sodium.

(iii) Briefly describe the different quantum numbers of an electron in an atom and explain the consequences of the Pauli Principle.

(iv) Into how many states does the electronic $3d$ level split in a strong external magnetic field (with respect to L and L_z). Explain your reasoning.

Question 4 (10 marks): Tunneling through a Barrier

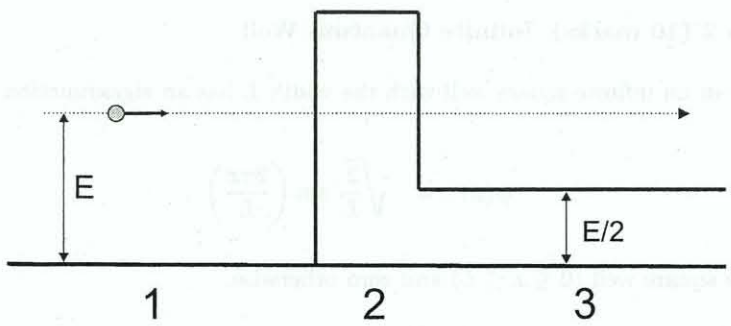
A particle with an energy E is tunneling through a potential barrier (see figure below). Note that such a tunneling process takes place for example at a metal-insulator-semiconductor thin film structure used in semiconducting industry.

(i) Schematically plot the form of the wavefunction expected in each of the 3 regions and give the functional forms of these wavefunctions (it is not required to give the precise mathematical expression of the final wavefunctions but the type of the waves). Explain the choice of the wavefunction for each region in a few words.

(ii) What are the mathematical boundary conditions for the waves when the particle enters and leaves the barrier?

(iii) Show how you would determine the various constants appearing in these wavefunctions.

(iv) If the potential energy in region 3 is exactly $E/2$, calculate the ratio of the wavelength in regions 1 and 3.



Question 5 (10 marks): The Hydrogen Atom

The ground state electronic wavefunction of the Hydrogen atom is

$$\psi(r) \quad = \quad a \cdot e^{-r/r_0}$$

where r_0 is the Bohr radius ($0.529 \cdot 10^{-10}$ m).

(i) Determine the normalization constant a.

(ii) Derive the expression for the probability density $P(r)$ as a function of r.

(iii) Sketch $\psi(r)$ and $P(r)$ and discuss the main differences between them.