THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS2040 QUANTUM PHYSICS

FINAL EXAMINATION
SESSION 1 – JUNE 2009

1. Time allowed – 2 hours

2. Reading time – 10 minutes

3. This examination paper has 4 pages.

4. Total number of questions – 5

5. Total number of marks – 65

6. All questions are NOT of equal value. Marks available for each question are shown in the examination paper.

7. Answer all questions.

8. ANSWER EACH QUESTION IN A SEPARATE BOOK.

9. Students are required to supply their own University approved calculator.

10. All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

11. This paper may be retained by the candidate.

p3

The following information is supplied as an aid to memory.

Planck's constant $h = 6.626 \times 10^{-34}$ Js

Fundamental charge unit $e = 1.60 \times 10^{-19}$ C

Speed of light (vacuum) $c = 3.0 \times 10^{8}$ m/s

Electron mass = 9.1×10^{-31} kg

Neutron mass = 1.675×10^{-27} kg

Proton mass = 1.672×10^{-27} kg

Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}

Angstrom (Å) = 1.0×10^{-10} m

Permittivity constant $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

Time-independent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x)$

Time-dependent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$

$\int \sin^2(bx)dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$

$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$

$\int x^2\sin^2(bx)dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin(2bx) - \frac{x\cos(2bx)}{4b^2}$

$\int_{-\infty}^{\infty} e^{-bx^2}dx = \sqrt{\frac{\pi}{b}}$

$\int_{1}^{\infty} \frac{1}{\sqrt{\pi}} e^{-t^2} dt \sim 0.08$

$\int_{0}^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

Bragg's law: $n\lambda = 2d\sin\theta$

Compton Shift: $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$

Energy levels in a Hydrogen-like atom with Z nucleons: $E_n = \frac{-me^4}{2\hbar^2(4\pi\varepsilon_0)^2}\frac{Z^2}{n^2} = -13.6eV \times \frac{Z^2}{n^2}$

Question 1 [10 Marks]

(a) Describe briefly the Compton effect and its significance for modern physics.

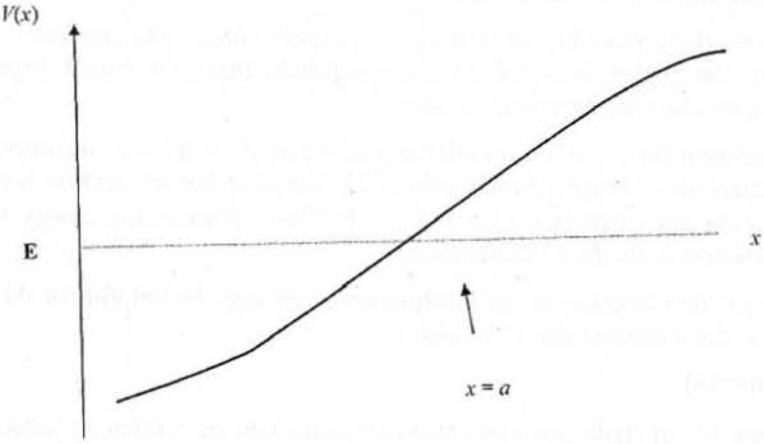
(b) If the de Broglie wavelength of an electron is equal to that of a proton, which has the larger speed? Explain why?

(c) In the Hydrogen atom, the light emitted when an electron goes from one energy state to another is not quite monochromatic. Using the uncertainty principle, estimate the wavelength spread in light emitted at a wavelength close to 6563Å, if the lifetime Δt for the transition is 10^{-8} s. (Hint: the derivative $dE/d\lambda$ might be helpful here.)

(d) A particle of mass m has a positional uncertainty equal to its de Broglie wavelength. Calculate the minimum fractional uncertainty in its velocity $\Delta v/v$.

Question 2 [10 marks]

A particle of energy E moves to the right in the slowly-increasing potential $V(x)$ shown below (i.e. $V(x)$ varies very slightly over one wavelength of the matter wave).



(a) Describe the motion of the particle using <u>classical mechanics</u> (i.e., the particle has no wave-like character).

(b) Describe the motion of the particle using quantum mechanics, and after discussing the functional form, wavelength and amplitude of the wavefunction in the relevant regions, sketch the wavefunction of the quantum particle as a function of x.

(c) Considering both the classical and quantum cases, can the particle be found to the right of $x = a$? (keep your answer brief, especially if you can reference earlier parts of the question).

Question 3 [13 Marks]

A particle of mass m moves in one-dimension under the influence of a potential $V(x)$. Suppose the particle is in an energy eigenstate $\psi(x) = (\gamma^2/\pi)^{1/4}\exp(-\gamma^2 x^2/2)$ with energy $E = \hbar^2\gamma^2/m$.

(a) Find the average position of the particle.

(b) Find the average momentum of the particle.

(c) The product of your answers for (a) and (b) is $< \hbar/2$. Is this a problem? Why/why not?

(d) Find the potential $V(x)$ and draw a sketch of $V(x)$ vs x.

Question 4 [18 Marks]

A 'Quantum Oscillator' is a potential which describes a particle that is subject to a restoring force proportional to the particle's displacement from equilibrium (c.f. Hooke's law for springs). The restoring force is $F = -kx$, the corresponding potential energy is $V(x) = \frac{1}{2}kx^2$ and the angular frequency is $\omega = (k/m)^{1/2}$.

(a) Write down the Schrödinger equation describing the motion of the particle of mass m in terms of ω and m (and not k).

(b) What can you say about the symmetry of the ground-state eigenfunction of this oscillator with regard to the centre of the potential well? Justify your answer.

(c) A trial eigenfunction for the ground-state is: $\phi(x) = C_o e^{-\alpha x^2}$

(i) Obtain an expression for the constant α in terms of ω and m.

(ii) Obtain an expression for the energy of the ground-state in terms of ω.

(iii) Calculate the normalization constant C_o.

(iv) Calculate the probability of finding the particle *outside* the classical region (i.e., outside the region bounded by the amplitude that you would expect for an equivalent classical harmonic oscillator)

(d) The virial theorem for a quantum oscillator states that $\langle T\rangle = \langle V\rangle$, i.e., the average kinetic energy T equals the average potential energy V. Suppose that an electron is confined in the ground-state such that $(\langle(x - \langle x\rangle)^2\rangle)^{1/2} = 10^{-10}$m. What is the energy required to excite the electron to the first excited state?

(n.b., some of the integrals on the information sheet may be helpful for this question. You will find the electron mass there also.)

Question 5 [14 marks]

(a) The complete set of hydrogen atom wavefunctions can be written as $\psi_{nlm}(r,\theta,\phi)$. The normalized wavefunction for the ground state of the hydrogen atom is given by $\psi_{100} = (\pi a^3)^{-1/2}e^{-r/a}$, where a is the Bohr radius.

(i) Explain the physical meaning of the quantum numbers n, l and m and state the values which they can take (in general).

(ii) Calculate the most probable value for the magnitude of r for this state, and;

(iii) The expectation value for the magnitude of r is $(3/2)a$. Resorting to a sketch graph if necessary, explain why the expectation value of r is greater than the most probable value of r.

(b) An energetic electron collides with a hydrogen atom and excites it directly from its ground state $n = 1$ to the state $n = 4$. Determine the minimum potential difference through which the electron must have been accelerated from rest.

(c) For the $n = 4$ state in the H atom, write down the largest allowed value of l.

(d) Lithium has an atomic number $Z = 3$. Two of its electrons have the quantum numbers $\{n, l, m_l, m_s\} = \{1,0,0,+1/2\}$ and $\{1,0,0,-1/2\}$. Write down the corresponding set of quantum numbers for the third electron in the lowest energy state of the Li atom. Briefly explain the factors motivating the set of quantum numbers