THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS2040 QUANTUM PHYSICS

MIDSESSION EXAMINATION
APRIL 2008

Time allowed – 55 minutes (start 13:05 end 14:00)

Total number of questions – 3

Total number of marks – 25

Answer ALL questions

The questions are NOT of equal value

This examination paper has 3 pages.

This paper may be retained by the candidate

Portable battery-powered electronic calculators (without alphabetic keyboards) may be used.

All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

The following information is supplied as an aid to memory.

Planck's constant $h = 6.626 \times 10^{-34}$ Js

Fundamental charge unit $e = 1.60 \times 10^{-19}$ C

Speed of light (vacuum) $c = 3.0 \times 10^{8}$ m/s

Electron mass = 9.1×10^{-31} kg

Neutron mass = 1.675×10^{-27} kg

Proton mass = 1.672×10^{-27} kg

Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}

Angstrom (Å) = 1.0×10^{-10} m

Permittivity constant $\varepsilon_o = 8.85 \times 10^{-12}$ Fm^{-1}

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

Time-independent Schrödinger Equation: $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x)$

Time-dependent Schrödinger Equation: $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\dfrac{\partial\psi(x,t)}{\partial t}$

$$\int \sin^2(bx)dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$$

$$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$$

$$\int x^2\sin^2(bx)dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin(2bx) - \frac{x\cos(2bx)}{4b^2}$$

$$\int_{-\infty}^{\infty} e^{-bx^2}dx = \sqrt{\frac{\pi}{b}}$$

$$\int_0^{\infty} x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Bragg's law: $n\lambda = 2d\sin\theta$

Compton Shift: $\Delta\lambda = \dfrac{h}{mc}(1-\cos\theta)$

Relativistic E-p relation: $E^2 = (pc)^2 + (mc^2)^2$ = Kinetic energy + Rest mass energy

Non-relativistic E-p relation: $E = p^2/2m$

Question 1 (10 Marks)

(a) Describe briefly the Davisson-Germer experiment and comment on its significance for Modern Physics.

(b) Calculate the wavelength of the most energetic X-rays produced when electrons of kinetic energy 50 keV strike a lead target.

(c) To what kinetic energy must an electron be accelerated so that its de Broglie wavelength is 25×10^{-12} m? What accelerating voltage would be required to achieve electrons with this kinetic energy? (Hint: you should be careful in deciding whether to do this calculation relativistically or non-relativistically. Explain which you choose to use and why.)

(d) A beam of electrons from an electron gun accelerated to a kinetic energy of 2.5 keV is incident on a sample of Nickel containing atomic planes spaced 2.15 Å apart. Calculate the smallest Bragg angle for which the intensity of the diffracted electrons will be a maximum.

Question 2 (8 Marks)

(a) A wavepacket for a particle has a small value of Δx, the uncertainty in position. Explain in terms of the formation of this wavepacket why the uncertainty in momentum will be large. (n.b., – simply saying that $\Delta p \Delta x \geq \hbar/2$, therefore $\Delta p \propto 1/\Delta x$ is not a satisfactory answer.)

(b) When a neutron decays it produces a proton, an electron and a particle called an electron-antineutrino ($\overline{\nu}_e$) whose mass you can take as zero (it turns out the mass is actually very small, and is currently an exciting research topic in physics). The beta decay process for the neutron is:

$$n \rightarrow p^+ + e^- + \overline{\nu}_e$$

(i) Using the uncertainty principle, estimate the energy of the antineutrino, if it is confined inside the neutron. Assume that the neutron is about the same size as a proton (i.e., 1×10^{-15} m).

(ii) If the energy of antineutrinos emitted from a beta decay process are measured, they are found to have a typical energy of around 1 MeV, What can you conclude from this information?

Question 3 (7 Marks)

A particle of mass m moves freely in one dimension in the region $0 \leq x \leq L$. The potential energy is zero within this region and is infinite outside the region. The wavefunction describing the particle has the form:

$$\psi_n(x,t) = C \sin\left(\frac{n\pi x}{L}\right) \exp\left(\frac{-iE_n t}{\hbar}\right)$$

(a) Evaluate the normalisation constant C. Why is normalisation important?

(b) Obtain an expression for the expectation value of the momentum $\langle p \rangle$. Comment on your answer.

(c) Calculate the probability for $n = 1$ of locating the particle in the first third of the box (i.e., between $x = 0$ and $x = L/3$). Give the numerical value.