THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS2040 QUANTUM PHYSICS

FINAL EXAMINATION
SESSION 1 – JUNE 2008

1. Time allowed – 2 hours

2. Reading time – 10 minutes

3. This examination paper has 5 pages.

4. Total number of questions – 5

5. Total number of marks – 50

6. All questions are NOT of equal value. Marks available for each question are shown in the examination paper.

7. Answer all questions.

8. The following materials will be provided by the Enrolment and Assessment Section: Calculators.

9. All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

10. This paper may be retained by the candidate.

The following information is supplied as an aid to memory.

Planck's constant $h = 6.626 \times 10^{-34}$ Js

Fundamental charge unit $e = 1.60 \times 10^{-19}$ C

Speed of light (vacuum) $c = 3.0 \times 10^{8}$ m/s

Electron mass = 9.1×10^{-31} kg

Neutron mass = 1.675×10^{-27} kg

Proton mass = 1.672×10^{-27} kg

Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}

Angstrom (Å) = 1.0×10^{-10} m

Permittivity constant $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

Time-independent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x)$

Time-dependent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$

$$\int \sin^2(bx)dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$$

$$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$$

$$\int x^2\sin^2(bx)dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin(2bx) - \frac{x\cos(2bx)}{4b^2}$$

$$\int_{-\infty}^{\infty} e^{-bx^2}dx = \sqrt{\frac{\pi}{b}}$$

$$\int_0^{\infty} x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Bragg's law: $n\lambda = 2d\sin\theta$

Compton Shift: $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$

Energy levels in a Hydrogen-like atom with Z nucleons: $E_n = \frac{me^4}{2\hbar^2(4\pi\varepsilon_0)^2}\frac{Z^2}{n^2}$

Question 1 [11 Marks]

(a) In one or two sentences, explain briefly the Photoelectric Effect.

(b) Light of frequency ν and relatively low intensity I shines on a metal cathode and the photoelectrons are collected by an anode at a positive potential V. The photocurrent i is reduced to zero by a reverse potential $V = -V_0$ (where $V_0 > 0$). The work function of the cathode is ϕ. Assuming that the quantum theory of light is correct, sketch 4 graphs showing the variation of:

(i) Photocurrent i with light intensity I for $V > 0$ and $\nu > \nu_0$, the cutoff frequency.

(ii) Photocurrent i with frequency ν for $V > 0$ and constant I.

(iii) Stopping potential V_0 with intensity I at constant ν.

(iv) Stopping potential V_0 with frequency ν at constant I.

You should identify key features (intercepts, gradients, etc. where possible).

(c) What are the three aspects of the photoelectric effect that cannot be explained by classical physics? How did Einstein's explanation of this effect overcome the problems with the classical interpretation?

(d) As a result of shining monochromatic light with wavelength λ on a metal surface, electrons are ejected with maximum kinetic energy T. If one changes the wavelength of the light to $\lambda/4$, electrons are ejected with a maximum kinetic energy $5T$. Obtain an expression for T in terms of λ.

Question 2 [8 Marks]

(a) In a region in space, a particle with zero total energy has a wave-function $\psi(x) = Ax \exp(-x^2/L^2)$ where L is a constant. Prove that the potential energy $U(x)$ is given by:

$$U(x) = \frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2} - 6\right)$$

(b) Consider an electron in a one-dimensional 'infinite wall' box of width L such that $V(x) = 0$ for $0 < x < L$ and $V(x) = \infty$ for $x \leq 0$ and $x \geq L$.

(i) If the wavefunction solutions take the form:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

Obtain an expression for the corresponding energy eigenvalues E_n.

(ii) In transitions between different energy levels, photons of various wavelengths are emitted or absorbed. It is found that the largest wavelength emitted is 450 nm. Calculate the width of the dot.

Question 3 [9 Marks]

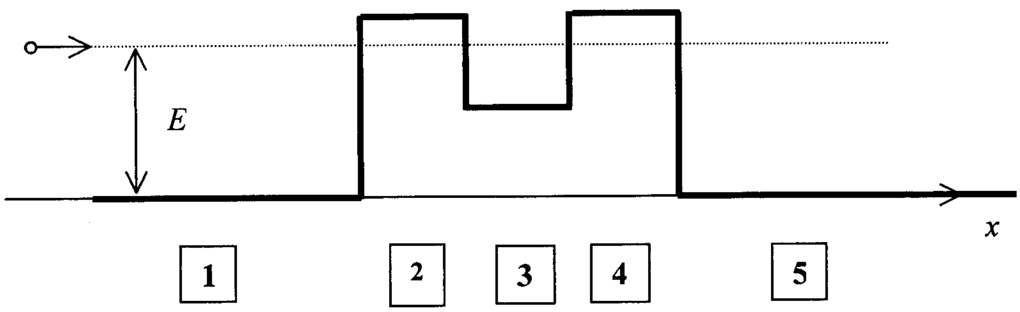
A beam of particles of energy E is fired from the left towards the symmetric potential shown in the figure below. Assume that regions 2 and 4 have a width sufficient for the particles to tunnel through, and that the width of region 3 is equal to a half-integer number of wavelengths $L_3 = n\lambda_3/2$.

(a) Sketch the form of the wavefunction expected in each of the 5 regions and give the functional forms of these wavefunctions (n.b., This does not require detailed mathematical expressions – just make sure it is clear what type of wave you are using. Ensure that you point out any significant features in your diagram that may not be obvious as you've drawn it e.g., how the wavelengths and amplitudes vary, etc.)

(b) Explain your choice of wavefunction for each region.

(c) Explain how you would attempt to determine the various constants appearing in these wavefunctions [do not try to do the mathematics].

(d) Assume you can gradually change the width L_3 or potential V_3, what will happen?



Question 4 [10 Marks]

(a) The ground state electronic wavefunction of the H atom is $\psi(r,\theta,\phi) = C\exp(-r/a_0)$ where a_0 is the first Bohr radius (0.529Å) and C is a constant. Obtain an expression for the normalization constant C.

(b) Calculate the most probable r value for this state.

(c) Sketch the probability distribution $P(r)dr$ as a function of radius for this state.

(d) Calculate the expectation value $\langle r \rangle$.

(e) The expectation value $\langle r \rangle$ differs significantly from the most probable r value, why is this so? Explain resorting to the definitions of these two values if necessary. If the expectation value doesn't coincide with the most probable r, then what does that tell you? Explain briefly.

Question 5 [12 Marks]

(a) State the three basic postulates of Bohr's model of the hydrogen atom.

(b) How does Bohr's model of the atom connect with de Broglie's idea that the electron is a wave?

(c) For the $n = 4$ state in (b) write down the largest allowed value of l.

(d) What is the magnitude of the orbital angular momentum L corresponding to the l value from (c)?

(e) Draw this angular momentum vector, showing all the possible projections of this vector onto the z-axis.

(f) Calculate the smallest angle that the angular momentum vector can make with the z-axis. Why can this angle not possibly be 0°?

(g) Lithium has an atomic number $Z = 3$. Two of its electrons have the quantum numbers $\{n, l, m_l, m_s\} = \{1,0,0,+1/2\}$ and $\{1,0,0,-1/2\}$. Describe briefly what physical quantity each of these quantum numbers represents.

(h) Write down the corresponding set of quantum numbers for the third electron in the lowest energy state of the Li atom. How does this depend on the presence/absence of an applied magnetic field?

(i) Assuming that Lithium ($Z = 3$) is a hydrogen-like atom and ignoring any screening effects due to other electrons in the atom, calculate its ionisation energy in eV.