PHYS 2040 – Midsession Exam '07'

1.

There was an earlier question, but
I dropped it to keep to time, hence the space.

Scroll
down.

V

1 (a). There are 3 aspects of the photoelectric effect that can't be explained by classical physics.

1. The maximum kinetic energy of the photons (as measured by the stopping potential V_0) is independent of intensity
2. The presence of a cutoff frequency, below which incident photons do not eject electrons from the metal surface
3. The lack of a time delay between light being shone on the metal surface and electrons being ejected, even at very low light intensities

(b) The key assumption is that light comes as discrete particles called photons which carry an energy dependent on their frequency $E = hf$. Additional to this is the assumption that photons sometimes behave more like particles than waves (ie. wave-particle duality).

(c). At the cutoff frequency, the stopping potential is zero, therefore K_{max} is also zero, thus $K_{max} = hf_0 - \phi = 0$

and $hf_0 = \phi$ so $\phi = hf_0 = \frac{hc}{\lambda_0} = \frac{6.626\times10^{-34} \times 3\times10^{8}}{650\times10^{-9}}$

$= 3.06\times10^{-19}\,J = \underline{1.91\,eV}$

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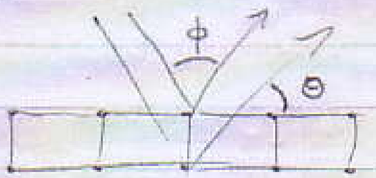
1 (c). A photon with $\lambda = 400nm$ have energy $E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{400\times10^{-9}}$

$$= 4.97\times10^{-19} J$$
$$= 3.106 eV.$$

$$\therefore K_{max} = E_{400nm} - \phi$$
$$= 3.106eV - 1.91eV$$
$$= 1.2eV. \ (= 1.91\times10^{-19} J).$$

2. This question needs a little thinking, and there's likely more than one way to do it, but the most obvious....

I'd start with the diffraction to work out what neutron wavelength you want.



$\phi = 163^\circ$ & $\phi + 2\theta = 180^\circ$

$\Rightarrow \theta = 8.5^\circ$.

Bragg's law $n\lambda = 2d\sin\theta$

$$\lambda = 2\times1.73\times10^{-10}\times\sin 8.5^\circ$$
$$= 5.11\times10^{-11} m \text{ or } 0.51 Å$$

Now the neutron has momentum $p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}}{5.11\times10^{-11}}$

$$= 1.3\times10^{-23} Kgm/s.$$

The neutrons are non-relativistic so $p = mv$ and so the neutron velocity $v = \frac{1.3\times10^{-23}}{1.675\times10^{-27}} \frac{Kgm/s}{Kg} = 7761 m/s.$

The neutron will travel the distance between the slots in a time

$$t = \frac{2m}{7761 m/s} = 2.58\times10^{-4} s.$$

In this time the shaft needs to rotate by 10°, so a full rotation needs to take $36\times2.58\times10^{-4} s = 9.29 ms$. $\therefore$ min rotational speed $= (9.29 ms)^{-1}$

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2. $\therefore$ min rotational speed $= (9.29\,\text{ms})^{-1}$

$$= 107.6 \text{ rev/sec}$$

$$= 6456 \text{ rpm.}$$

n.b. you could run at integer multiples of the above speed and still select out the same beam speed, hence the request for the minimum speed

3 (a). $\bar{x} = \int_{-\infty}^{\infty} \psi^* x \psi \, dx = \int_0^L \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right) x \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right) dx$

$$= \frac{2}{L} \int_0^L x \sin^2\left(\frac{2\pi x}{L}\right) dx.$$

we need to change variables here so we can use a known result.

use $\Theta = \frac{2\pi x}{L} \Rightarrow \frac{d\Theta}{dx} = \frac{2\pi}{L} \Rightarrow dx = \frac{L\,d\Theta}{2\pi}$ and if $x = L$ $\Theta = 2\pi$

$\Rightarrow x = {}^{L\Theta}/_{2\pi}$ $\qquad x = 0$ $\Theta = 0$.

so $\bar{x} = \frac{2}{L} \int_0^{2\pi} \frac{L\Theta}{2\pi} \sin^2\Theta \frac{L\,d\Theta}{2\pi}$

$$= \frac{2}{L}\frac{L^2}{(2\pi)^2} \int_0^{2\pi} \Theta \sin^2\Theta \, d\Theta$$

now $\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}.$

$$\therefore \bar{x} = \frac{2L^2}{4\pi^2 L}\left[\frac{\Theta^2}{4} - \frac{\Theta \sin 2\Theta}{4} - \frac{\cos 2\Theta}{8}\right]_0^{2\pi}$$

$$= \frac{L}{2\pi^2}\left[\frac{4\pi^2}{4} - \frac{2\pi \cancel{\sin(4\pi)}^{0}}{4} - \frac{\cos 4\pi^{1}}{8} - \frac{0^2}{4} + \frac{\cancel{0 \sin 0}^{0}}{4} + \frac{\cos 0^{1}}{8}\right]$$

$$= \frac{L}{2\pi^2}\left[\pi^2 - \frac{1}{8} + \frac{1}{8}\right]$$

$$= \frac{L}{2}.$$

P.T.O.

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3(b) We need the probability of finding the particle in the range $L/2 \pm 0.01L \Rightarrow x_f = 0.51L \quad x_i = 0.49L$.

$$P(x) = \int_{x_i}^{x_f} \psi^* \psi \, dx$$

$$= \frac{2}{L} \int_{x_i}^{x_f} \sin^2\left(\frac{2\pi x}{L}\right) dx.$$

two routes: use a standard result $\int \sin^2(bx)\,dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$

or use $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ and solve that integral

using the result

$$P(x) = \frac{2}{L}\left[\frac{x}{2} - \sin\left(\frac{4\pi x}{L}\right) \times \frac{L}{8\pi}\right]_{x_i}^{x_f}$$

$$= \frac{1}{L}\left[x - \frac{L}{4\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{x_i}^{x_f}$$

so $$P(0.49L \to 0.51L) = \frac{1}{L}\left[0.51L - \frac{L}{4\pi}\sin\left(\frac{4\pi \times 0.51L}{L}\right) - 0.49L + \frac{L}{4\pi}\sin\left(\frac{4\pi \times 0.49L}{L}\right)\right]$$

$$= \frac{1}{L}\left[0.02L - \frac{L}{4\pi}\left(\sin(2.04\pi) - \sin(1.96\pi)\right)\right]$$

$$= 0.02 - \frac{1}{4\pi}(0.2507)$$

$$= 0.00005$$

3(c) Same answer but $L/4 \pm 0.01L \Rightarrow x_f = 0.26L \quad x_i = 0.24L$

$$P(0.24L \to 0.26L) = \frac{1}{L}\left[0.26L - \frac{L}{4\pi}\sin\left(\frac{4\pi \times 0.26L}{L}\right) - 0.24L + \frac{L}{4\pi}\sin\left(\frac{4\pi \times 0.24L}{L}\right)\right]$$

$$= \frac{1}{L}\left[0.02L - \frac{L}{4\pi}\left(\sin(1.04\pi) - \sin(0.96\pi)\right)\right]$$

$$= 0.02 - \frac{1}{4\pi}(-0.2507)$$

$$= 0.0399.$$

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3(d). This is quite easy. if you look at the wavefunction graphically, when you take $\psi^*\psi$ you should get two maxima located at $L/4$ & $3L/4$ hence the high probability of being there. The wavefunction has a node at $L/2$ and so the probability of finding it there should be almost zero. The expectation value, as a weighted average, will just be the symmetry point for the wavefunction, hence $L/2$. This example highlights that the expectation value of x is not necessarily the most likely place to find the particle.