THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS2040 QUANTUM PHYSICS

FINAL EXAMINATION
SESSION 1 – JUNE/JULY 2007

1. Time allowed – 2 hours

2. Reading time – 10 minutes

3. This examination paper has 4 pages.

4. Total number of questions – 5

5. Total number of marks – 50

6. All questions are NOT of equal value. Marks available for each question are shown in the examination paper.

7. Answer all questions.

8. The following materials will be provided by the Enrolment and Assessment Section: Calculators.

9. All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

10. This paper may be retained by the candidate.

PPS 60

The following information is supplied as an aid to memory.

Planck's constant $h = 6.626 \times 10^{-34}$ Js

Fundamental charge unit $e = 1.60 \times 10^{-19}$ C

Speed of light (vacuum) $c = 3.0 \times 10^{8}$ m/s

Electron mass = 9.1×10^{-31} kg

Neutron mass = 1.675×10^{-27} kg

Proton mass = 1.672×10^{-27} kg

Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}

Angstrom (Å) = 1.0×10^{-10} m

Permittivity constant $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

Time-independent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x)$

Time-dependent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$

$$\int \sin^2(bx)dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$$

$$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$$

$$\int x^2\sin^2(bx)dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin(2bx) - \frac{x\cos(2bx)}{4b^2}$$

$$\int_{-\infty}^{\infty} e^{-bx^2}dx = \sqrt{\frac{\pi}{b}}$$

$$\int_0^{\infty} x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$

$\sin(2\theta) = 2\sin\theta\cos\theta$

Bragg's law: $n\lambda = 2d\sin\theta$

Compton Shift: $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$

Question 1 [10 Marks]

(a) In a few sentences, discuss the Compton effect (i.e., the experiment and the result) and its implications for the development of the quantum theory.

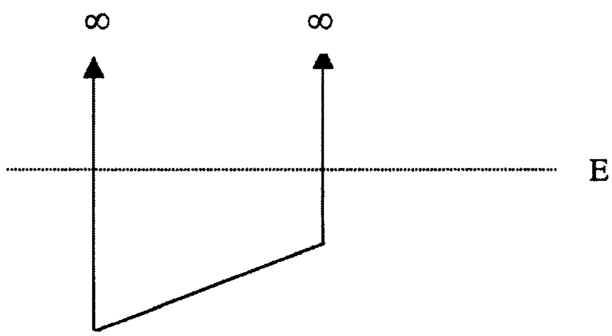
(b) Calculate the scattering angle θ of a photon in a Compton process such that the photon loses 10% of its initial energy $E = 200$ keV during the process.

(c) A wavepacket for a particle has a small value of Δx, the uncertainty in position. Briefly explain in terms of the formation of this wavepacket why the uncertainty in momentum Δp will be large (n.b., simply saying $\Delta x \Delta p \geq \hbar/2$ therefore $\Delta p \propto 1/\Delta x$ is not the answer).

(d) An electron is confined in an atom of diameter 0.1 nm. Calculate the uncertainty of the electron's momentum. Is this consistent with the typical binding energy of electrons in atoms? Explain your answer.

Question 2 [10 Marks]

(a) An electron is in an infinite potential well whose bottom is tilted, as shown in the diagram. Draw a wavefunction showing qualitatively the variation of both amplitude and wavelength as a function of position in the well. Write a short explanation for the behaviour of your wavefunction. (Hint: Do **not** attempt to solve the Schrödinger equation for this well, use your knowledge of how wavefunctions behave to develop an answer).



(b) A particle is described by the wavefunction $\psi(x) = A(L^2 - x^2)e^{ikx}$ for $-L \leq x \leq L$ and $\psi(x) = 0$ elsewhere.

(i) Normalise this wavefunction

(ii) Calculate the expectation value of the x-component of the momentum (show all your working).

Question 3 [8 Marks]

A 'Quantum Oscillator' is a potential that describes a particle that is subject to a restoring force proportional to the particle's displacement from equilibrium. The corresponding potential energy is $V(x) = \frac{1}{2}Kx^2$, and the angular frequency is $\omega = (K/m)^{1/2}$.

(a) Write down the Schrödinger equation describing the motion of the particle of mass m in terms of ω and m (and not K).

(b) What can you say about the symmetry of the ground-state eigenfunction of this oscillator with regard to the centre of the potential well? Justify your answer.

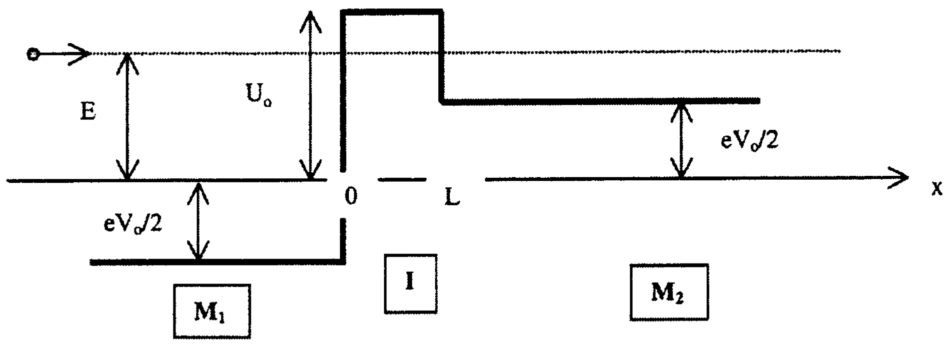
(c) A trial wavefunction for the ground-state is $\psi(x) = C_0 \exp(-\alpha x^2)$:

(i) Obtain an expression for the constant α in terms of ω and m.

(ii) Obtain an expression for the energy of the ground state in terms of ω.

Question 4 [9 Marks]

An approximate representation of the potential seen by an electron in a metal-insulator-metal (M_1-I-M_2) sandwich is shown below. A potential difference V_o exists between the two metals.



(a) Write down the time-independent Schrödinger equation for the wavefunction of an electron of energy E for each of the three regions M_1, I and M_2.

(b) An electron of energy E from metal M_1 is incident on the barrier. Write down the form of the electron wavefunction in each region.

(c) Sketch the form of the wavefunction in each region. Briefly discuss the relative wavelength and amplitude of the wavefunction in regions M_1 and M_2.

Question 5 [13 Marks]

(a) State the three basic postulates of Bohr's model of the hydrogen atom, and briefly discuss the problems associated with the Rutherford model that Bohr's model was attempting to overcome.

(b) Show that the radius and energy of the ground state electron orbit of the Bohr atom are given by: $r_o = \frac{\varepsilon_o h^2}{\pi m e^2}$ and $E_o = -\frac{m e^4}{8 \varepsilon_o^2 h^2}$

(c) When Quantum Mechanics is applied to the hydrogen atom one finds that 4 quantum numbers are needed to describe the state of the electron. What are these quantum numbers and what physical quantities are quantised by them? You may use a diagram to show the quantisation and orientation of some of these quantities if you wish.

(d) What values can the quantum numbers in (c) take if one considers the fourth electron shell

(e) Describe briefly (i.e., in 2-3 sentences) the Stern-Gerlach experiment and its significance for quantum physics.