THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – JUNE 2013

PHYS2010 – MECHANICS

PHYS2120 – MECHANICS & COMPUTATIONAL PHYSICS (Mechanics Paper)

PHYS9483 ADVANCED THEORETICAL PHYSICS 1

Time allowed – 2 hours

Total number of questions – 4

Answer ALL FOUR questions.

The questions are of equal value.

This paper may be retained by the candidate.

Candidates may not bring their own calculators.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{c}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$
$$\theta = \pm\int \frac{\left(L/r^2\right)dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$

Lagrangian

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Question 1

A particle of mass m moves in a two-dimensional plane (x-y) under the influence of an isotropic harmonic potential:

$$V(r) = \frac{1}{2}kr^2$$

(a) Use either Newton's laws of motion or the Lagrangian method to determine the equations of motion for this system.

(b) Solve the two equations of motion as a function of time.

(c) Derive the path equation for the motion of the particle.

(Hint: You need to eliminate time from the two equations so as to produce a path equation G(x,y)=0. One equation contains a term $\cos(\omega t+\Phi_A)$ while a second contains $\cos(\omega t+\Phi_B)$ substitute the latter term with $\cos(\{\omega t+\Phi_A\}-\Delta)$ where $\Delta=\Phi_B-\Phi_A$. Use the double angle relationship to eliminate time and hence derive the orbit equation.)

(d) Describe the types of path that will be followed by the particle under the influence of the isotropic harmonic potential.

The above potential is replaced by a non-isotropic harmonic potential:

$$V(x,y) = \frac{1}{2}\left(k_1 x^2 + k_2 y^2\right)$$

where the two spring constants k_1 and k_2 are not equal.

(e) Describe the motion of a particle moving under the influence of this non-isotropic force.

(f) Sketch the general orbit for arbitrary spring constants.

Question 2

Part A

Two particles of mass m and M interact via a force that is directed along the line connecting the two particles.

(a) Show that the motion of this physical system can be reduced to the motion of a single (hypothetical) particle in a central force.

(b) Discuss the implications of the non-conservation of linear momentum in the solution of the central force problem for the two body system.

Part B

A particle moves in a central field with potential $V(r)$. The velocity in plane polar coordinates is given by:

$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\theta}$$

(c) Write an expression for the total energy of the particle in the central field.

(d) Use the conservation of angular momentum to reduce this equation to be a function of radius and radial velocity (i.e. eliminate terms dependent on θ and its time derivatives).

The potential has the following form:

$$V(r) = -\frac{k}{2r^2}$$

(e) Write an equation for the effective radial potential produced by this potential function.

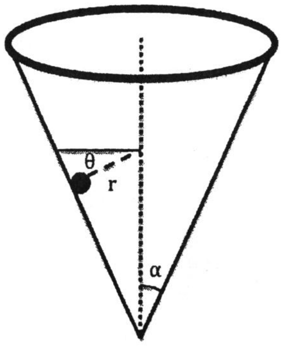
The form of the equation for the effective potential produces three distinct curves that depend on the values of the parameters: m, the particle mass; h (or l), the angular momentum per unit mass; and k, the force constant.

(f) Sketch the three possible effective potential curves for this central force as a function of radius. Label each curve with the conditions on m, h (or l) and k that are appropriate to the curve.

(g) What type of motion will be experienced by the particle in each of the three cases?

Question 3

PART A



A particle is confined to slide on the inside surface of a frictionless cone. The cone is fixed to a table so that its axis is perpendicular to the table. The cone half angle is α. The coordinates of the particle are given by r, the distance from the axis of the cone and θ the angle around the axis. Gravity acts on the particle.

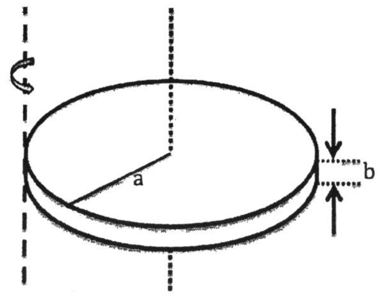
(a) Derive an expression for the total kinetic energy of this system. (Hint: you must consider motion around the cone as well as motion perpendicular to this up the side of the cone.)

(b) Derive an expression for the Lagrangian of this system.

(c) Use Lagrange's equations to find the equations of motion for this system.

(d) Show that the angular momentum of the particle is conserved.

PART B



A solid disc of mass, M, radius, a, and thickness, b, rotates about an axis that touches its rim and is parallel to its symmetry axis.

(e) Derive an expression for the moment of inertia of the disc about its rotation axis.

Question 4

A linear triatomic molecule can be modelled using classical mechanics as comprising a central mass, M, coupled to two smaller masses, m (one on each side). The coupling between the central mass and each of the two smaller masses can be modelled as a spring of stiffness K.

For simplicity, only consider motion along the axis of this molecule, i.e. one dimensional motion.

(a) How many degrees of freedom does this system have assuming it is confined to motion along its axis (i.e. one dimension)?

(b) Select a set of generalised coordinates to describe the state of this model for a linear triatomic molecule. Show your coordinates on a sketch of the molecule.

(c) Write expressions for the kinetic energy T and the potential energy V for this molecule and hence write an expression for the Lagrangian.

(d) Derive the equations of motion for this molecule.

(e) Derive the secular equation for the system.

(f) Solve the secular equation and find the normal frequencies of the system.

(g) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.