THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – JUNE 2012

PHYS2010 – MECHANICS

PHYS2120 – MECHANICS and COMPUTATIONAL PHYSICS Paper 1

Time allowed – 2 hours

Total number of questions – 4

Answer ALL FOUR questions.

The questions are of equal value.

This paper may be retained by the candidate.

Candidates may not bring their own calculators.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{c}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$
$$\theta = \pm\int \frac{\left(L/r^2\right)dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$

Lagrangian

$$L = T - U$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Question 1

For the damped harmonic oscillator, the equation of motion can be written as:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(a) explain all terms in this equation.

This results in three distinct types of solutions that represent three types of physical behaviour: overdamping, critical damping and underdamping.

(b) Explain each of these modes of physical behaviour. Use sketches to illustrate the motion of the particle in each case.

The above oscillator is driven by a harmonic driving force given by:

$$F(t) = F_o e^{i\omega t}$$

On solving the equations of motion for this driven damped harmonic oscillator, one finds that the amplitude, $A(\omega)$, as a function of frequency is given by:

$$A(\omega) = \frac{F_o/m}{\left[(\omega_o^2 - \omega^2)^2 + 4\gamma^2\omega^2\right]^{\frac{1}{2}}}$$

(c) Explain the meaning of all the symbols in the equation for $A(\omega)$.
(d) Derive an equation for the resonant frequency of the system.
(e) Sketch $A(\omega)$ as a function of ω for various values of γ. Be sure to plot and label curves where the behaviour of the system changes.

The phase shift in the driven damped harmonic oscillator is given by:

$$\tan(\phi(\omega)) = \frac{2\gamma\omega}{\omega_o^2 - \omega^2}$$

(f) Explain the meaning of this equation.
(g) Plot ϕ as a function of ω for several values of γ. Label all important parameters on your graph (common points, asymptotes, inflection points etc).

Question 2

Part A

Two particles of mass m and M interact via a force that is directed along the line connecting the two particles.

(a) Show that the motion of this physical system can be reduced to the motion of a single (hypothetical) particle in a central force.

(b) Discuss the implications of the non-conservation of linear momentum in the solution of the central force problem for the two body system.

Part B

A particle moves in a central field with potential $V(r)$. The velocity in plane polar coordinates is given by:

$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\theta}$$

(c) Write an expression for the total energy of the particle in the central field.

(d) Use the conservation of angular momentum to reduce this equation to be a function of radius and radial velocity (i.e. eliminate terms dependent on θ and its time derivatives).

The potential has the following form:

$$V(r) = -\frac{C}{3r^3}$$

(e) Write an equation for the effective radial potential produced by this potential function.

(f) Find the maximum value for the effective potential.

(g) Sketch the effective radial potential.

(h) Discuss the types of motion for a particle in this central field.

Question 3

A wedge of mass M with an angle ϕ is free to slide on a frictionless horizontal table. A solid ball of radius a and mass m is placed on the slope of the wedge. The contact between the ball and the wedge is perfectly rough.

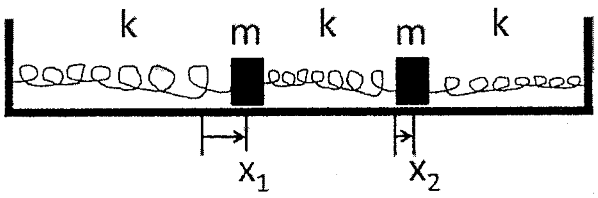
Note that the moment of inertia for a solid ball (mass m and radius a) is given by:

$$I = \frac{2}{5}ma^2$$

(a) Derive an expression for the total kinetic energy of this system.
(b) Derive an expression for the Lagrangian of this system.
(c) Use Lagrange's equations to find the equations of motion for this system.
(d) Derive an analytic expression for the horizontal acceleration of the wedge with respect to the tabletop.
(e) Discuss the dependence of the acceleration of the wedge on the radius of the sphere.

Question 4

Two equal masses m can move without friction on a horizontal table. The masses are connected by three springs each with elastic constant k. One spring connects the masses to each other while the other two each connect a single mass to its adjacent wall.



The masses can move only along one line as it is shown in Figure. The coordinates x_1 and x_2 denote displacements of each mass from the corresponding equilibrium positions.

(a) Write down the Lagrangian for this system in terms x_1 and x_2 and hence derive the equations of motion.

(b) Derive the secular equation for the system.

(c) Solve the secular equation and find the normal frequencies of the system.

(d) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.