THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION TEST – APRIL 2011

PHYS2010 – MECHANICS

Time allowed – 50 minutes

Total number of questions – 4

Answer ALL questions.

Answer ALL parts

The questions are of equal value.

This paper may be retained by the candidate.

NO calculators are to be used for this paper.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{b}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Lagrangian

$$L = T - U$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Question 1

A simple harmonic oscillator is described by the equation of motion:

$$m\ddot{x} + kx = 0$$

(a) Write a general form for the solution to this equation of motion.
(b) What is the natural frequency of this oscillator?

This oscillator is altered by the addition of a damping term:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(c) Under what conditions will this oscillator display underdamped behaviour?
(d) Derive an expression for the frequency of this oscillator when it is underdamped.

The damped oscillator is now driven by a harmonic driving force given by:

$$F = F_o \cos \omega t$$

(e) Under what conditions will this damped, driven harmonic oscillator display resonance? (i.e for what values of c, the damping coefficient)
(f) How will the resonant frequency compare with that of the undriven, damped harmonic oscillator?

Question 2

For an isotropic central force, the radial equation of motion is given by:

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = f(r)$$

Using the change of variable:

$$r = \frac{1}{u}$$

and the result:

$$\ddot{r} = -h^2u^2\frac{d^2u}{d\theta^2}$$

where h is the angular momentum per unit mass.

(a) Derive the path equation for the orbit:

$$\frac{d^2u}{d\theta^2} + u = -\frac{1}{mh^2u^2}f\left(u^{-1}\right)$$

(b) Hence (or otherwise) derive the orbit equation for an inverse square force.

(c) Derive a general solution for the orbit equation of the inverse square force.

(d) What types of orbit (geometric shapes) are described by this orbit equation?

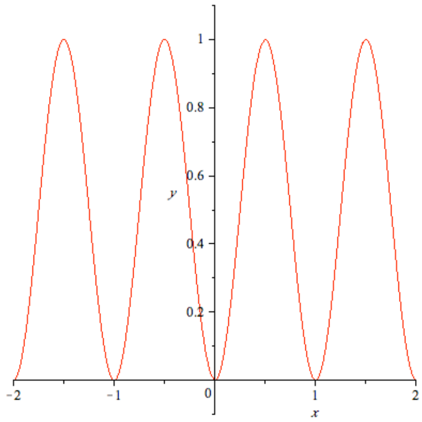
Question 3

PART A

A particle of mass m moves along the x axis under the influence of the potential:

$$V(x) = 1 - \cos^2(\pi x)$$

which is sketched below.



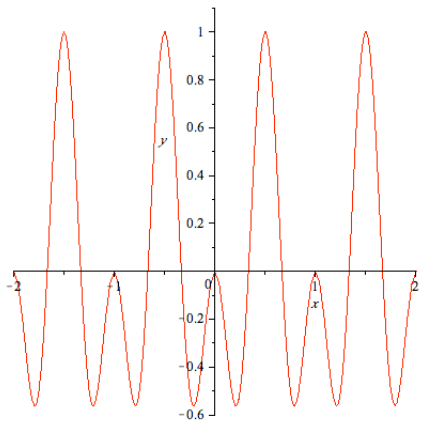
(a) Sketch (roughly) the velocity phase space portrait of the system.
(b) Indicate the separatrix in your sketch.
(c) From the velocity phase space portrait, discuss the motion of the particle in the system as a function of energy and initial conditions.

PART B

The above potential is modified as per the equation:

$$V(x) = 1 - \cos^2(\pi x) - \sin^2(2\pi x)$$

which is sketched below.



(d) Sketch (roughly) the new velocity phase space portrait for the system.
(e) Indicate the separatrixes in your sketch (there are three per repeat!).

Question 4

In plane polar coordinates, the velocity is given by:

$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$$

A particle of mass m moves under the action of an isotropic central force:

$$\mathbf{f}(r) = f(r)\hat{\mathbf{r}}$$

(a) Write an expression for the Lagrangian for the particle in the isotropic central force using plane polar coordinates.

(b) Use the Lagrangian to derive two equations of motion for this system.

The Lagrangian does not explicitly depend on the angular variable θ.

(c) Show that this implies that the angular momentum of the particle is conserved in the presence of the central force.

(d) Use the conservation of angular momentum to derive radial equation of motion that depends only on r and its total time derivatives ($\dot{r}$ and/or $\ddot{r}$).