THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2010

PHYS2010

Mechanics

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions are of equal value

Candidates must supply their own, university approved, calculator.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Candidates may keep this paper.

Examination 2010
Mechanics PHYS2010
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The formula sheet.

Damped Harmonic Motion

$$m\ddot{x} + \alpha\dot{x} + kx = 0$$
$$x = De^{rt}$$
$$r = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$$
$$\lambda = \frac{\alpha}{2m}$$
$$\omega^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + \alpha\dot{x} + kx = F_0\cos(\gamma t)$$
$$x = A\cos(\gamma t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{(\gamma^2 - \omega^2)^2 + 4\lambda^2\gamma^2}}$$
$$\tan\varphi = \frac{2\lambda\gamma}{\omega^2 - \gamma^2}$$
$$\gamma_r^2 = \omega^2 - 2\lambda^2$$

Central field

$$U_{eff}(r) = \frac{M^2}{2mr^2} + U(r)$$
$$\varphi = \pm\int \frac{\frac{M}{r^2}dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

Lagrangian

$$\mathcal{L} = T - U\ .$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Symmetric top

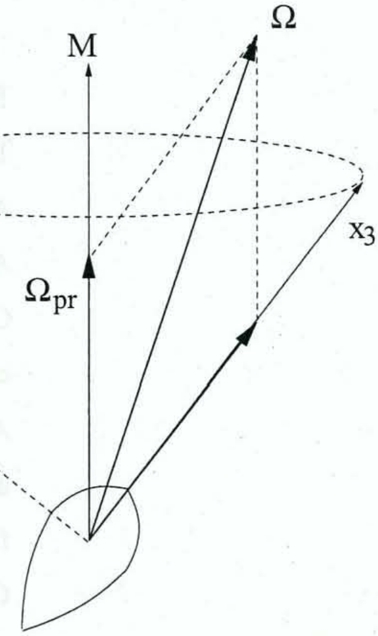

FIG. 1:

$$\Omega_3 = \frac{M_3}{I_3}$$

$$\Omega_{pr} = \frac{M}{I_1}$$

Question 1

Consider an overdamped, $\lambda = 2\omega$, forced harmonic oscillator

$$\ddot{x} + 4\omega\dot{x} + \omega^2 x = F/m \ .$$

The external force has frequency $\gamma = \omega$

$$F = F_0 \cos(\omega t) = Re\left\{F_0 e^{i\omega t}\right\} \ .$$

a. (3 marks) Derive the free (homogeneous) solution for the oscillator. The solution must depend on two arbitrary constants. Compare your result with the general formula presented in the formula sheet.

b. (3 marks) Derive the forced (particular) solution for the oscillator. The solution has no arbitrary constants. Compare your result with the general formula presented in the formula sheet.

c. (4 marks) Write down the general solution that is a superposition of the free (homogeneous) and the forced (particular) solutions and hence find $x(t)$ if at $t = 0$ the oscillator is in rest

$$x(t = 0) = 0 \ ,$$

$$\dot{x}(t = 0) = 0 \ .$$

Question 2

A block of mass m can slide horizontally along a rail without friction. Two pendulums are attached to the block. Each pendulum consists of a massless rod and a pointlike object of the *same* mass m. The length of the first pendulum is l, and the length of the second pendulum is $2l$. The objects can move only in the plane of the figure.

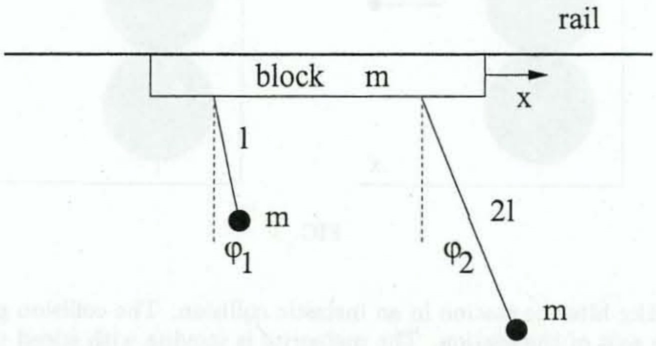

FIG. 2:

The dynamic variables for the system are the angles φ_1, φ_2 and the block displacement x. Assume that the angles are small, $\varphi_1, \varphi_2 \ll 1$, so the leading approximation is always sufficient, $\sin\varphi \approx \varphi$, $\cos\varphi \approx 1 - \varphi^2/2$. The gravitational field g is directed down.

a. (5 marks) Derive the Lagrangian of the system in terms of φ_1, φ_2, x and their time derivatives.

b. (5 marks) Using the Lagrangian derive the equations of motion of the system and hence show that they are of the following form

$$3\ddot{x} + l\ddot{\varphi}_1 + 2l\ddot{\varphi}_2 = 0$$
$$l\ddot{\varphi}_1 + \ddot{x} + g\varphi_1 = 0$$
$$2l\ddot{\varphi}_2 + \ddot{x} + g\varphi_2 = 0$$

Question 3

Consider the system from question **2**.

a. (3 marks) Derive the secular equation for the system.

b. (3 marks) Solve the secular equation and find the normal frequencies of the system.

c. (4 marks) Find the normal modes.

Question 4

A space station consists of two spherical shells of radius $r = 2$meters and mass $M = 1000$kg each. Connection between the shells is rigid. The moment of inertia of each shell is $I = \frac{2}{3}Mr^2$. Initially the space station is in rest. A

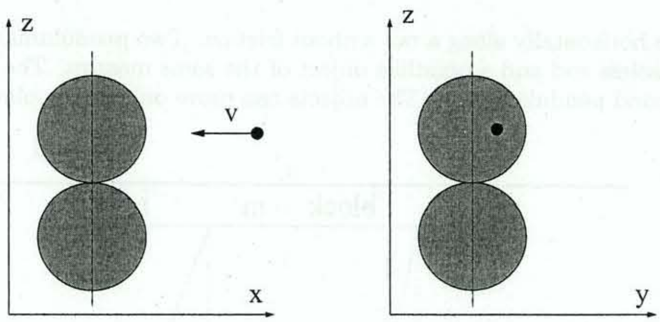

FIG. 3:

meteorite of mass $m = 0.01$kg hits the station in an inelastic collision. The collision geometry is shown in Fig.3. The z-axis is directed along the axis of the station. The meteorite is moving with speed $v = 10$km/sec along the straight line parallel to the x-axis. In the xz-projection the line is at the same level as the center of the top sphere, Fig.3 left. In the yz-projection the line hits the sphere at distance $r/2 = 1$ meter from the center of the top sphere, Fig.3 right.

a. (1 mark) In 1 - 2 sentences describe the motion of the station after collision. Describe it by appropriate diagrams.

b. (1 mark) Find the linear velocity of the station after collision.

c. (2 marks) Find the angular momentum of the system.

d. (3 marks) Find the angular velocity of the station precession after the collision.

e. (3 marks) Find the angular velocity of the station rotation around its axis after the collision.