THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION TEST – APRIL 2009

PHYS2010 – MECHANICS

Time allowed – 50 minutes

Total number of questions – 4

Answer ALL questions.

Answer ALL parts

The questions are of not of equal value.

This paper may be retained by the candidate.

NO calculators are to be used for this paper.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{b}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Lagrangian

$$L = T - U$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Question 1 (8 marks)

A simple harmonic oscillator is described by the equation of motion:

$$m\ddot{x} + kx = 0$$

(a) Write a general form for the solution to this equation of motion.
(b) What is the natural frequency of this oscillator?

This oscillator is altered by the addition of a damping term:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(c) Under what conditions will this oscillator display underdamped behaviour?
(d) Derive an expression for the frequency of this oscillator when it is underdamped.

The damped oscillator is now driven by a harmonic driving force given by:

$$F = F_o \cos \omega t$$

(e) Under what conditions will this damped, driven harmonic oscillator display resonance? (i.e for what values of c, the damping coefficient)
(f) How will the resonant frequency compare with that of the undriven, damped harmonic oscillator?

Question 2 (4 marks)

A force $F(\dot{x},t)$ has the form:

$$F(\dot{x},t) = f(\dot{x})g(t)$$

(a) Show that the equation of motion for this force $F(\dot{x},t)$ is integrable.

A second force $F(x,\dot{x})$ has the form:

$$F(x,\dot{x}) = f(x)g(\dot{x})$$

(b) Show that the equation of motion for this force $F(x,\dot{x})$ is integrable.

Question 3 (2 marks)

A force is given by:

$$\mathbf{F}(x,y,z) = \left(5xy + z^3\right)\mathbf{i} + \left(x^2\right)\mathbf{j} + \left(3xz^2\right)\mathbf{k}$$

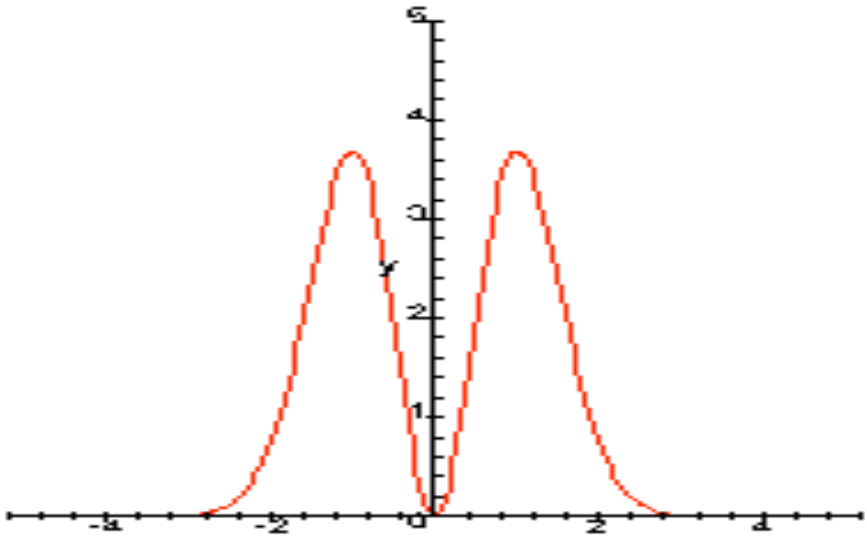
(a) Determine whether this force is conservative.

Question 4 (6 marks)

A particle of mass m moves along the x axis under the influence of the potential

$$V(x) = 10.x^2 e^{-x^2}$$

which is sketched below.



(a) Sketch (roughly) the velocity phase space portrait of the system
(b) Indicate the separatrix
(c) From the velocity phase space portrait, discuss the motion of the particle in the system as a function of energy and initial conditions