THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2009

PHYS2010

MECHANICS

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions ARE of equal value

Answer question 1 in one answer book.

Answer questions 2, 3 and 4 in the other book.

Candidates are required to supply their own, university approved calculator

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Candidates may keep this paper.

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Examination 2009
Mechanics PHYS2010
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The questions are of equal value.
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The formula sheet.

Central field

$$U_{eff}(r) = \frac{M^2}{2mr^2} + U(r)$$
$$\varphi = \pm \int \frac{\frac{M}{r^2}dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$
$$t = \pm m \int \frac{dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

Lagrangian

$$\mathcal{L} = T - U \; .$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Symmetric top

$$\Omega_3 = \frac{M_3}{I_3}$$

$$\Omega_{pr} = \frac{M}{I_1}$$

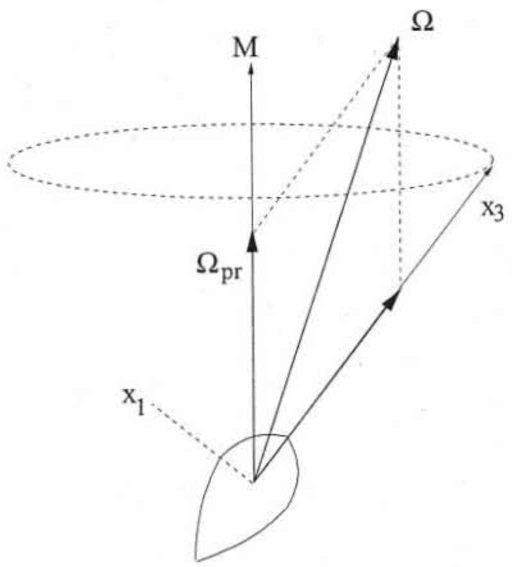

FIG. 1:

Question 1

A simple harmonic oscillator with no damping is described by the equation of motion:

$$m\ddot{x} + kx = 0$$

where m is the mass of the particle and k is the spring constant for the elastic restoring force.
Such a simple harmonic oscillator can be coupled to an external harmonic driving force:

$$F_{ext} = F_0 \exp(i\omega t)$$

(i) What is the equation of motion for this simple harmonic oscillator driven by this harmonic driving force in the absence of damping?

This oscillator is likely to behave badly when the frequency of the driving force is set to the natural frequency of the free oscillator.
For frequencies that are not equal to this natural frequency, the solution of the equation of motion for the driven oscillator will have the form:

$$x(t) = A\exp[i(\omega t - \phi)]$$

(ii) Substitute this trial solution into your equation of motion to derive a complex equation relating A, ϕ and ω, where your equation is independent of time.

(iii) By separating this equation into real and complex components (or otherwise), derive an expression for $\phi(\omega)$ and hence determine the allowed values of ϕ.

(iv) Using the allowed values of ϕ plus the real and/or complex components of the above equation, derive an expression for $A(\omega)$.

(v) Sketch both $A(\omega)$ and $\phi(\omega)$, labeling special values of ϕ and ω on your plot.

(vi) How does the oscillator behave when the driving force is set to the natural frequency of the simple harmonic oscillator?

Question 2

A block of mass m can slide horizontally along a rail without friction. Two identical pendulums are attached to the block. Each pendulum consists of a massless rod of a length l and a pointlike object of the *same* mass m. The objects can move only in the plane of the figure. Dynamic variables of the system are angles φ_1, φ_2 and

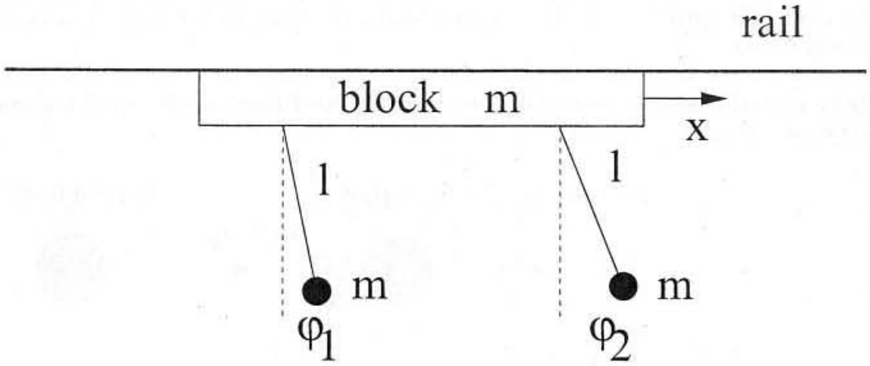

FIG. 2:

the block displacement x. Assume that the angles are small, $\varphi_1, \varphi_2 \ll 1$, so leading approximation is always sufficient, $\sin\varphi \approx \varphi$, $\cos\varphi \approx 1 - \varphi^2/2$. The gravitational field g is directed down.

a) Derive Lagrangian of the system in terms of φ_1, φ_2, x and their time derivatives.

b) Using the Lagrangian derive equations of motion of the system and hence show that they are of the following form

$$\begin{aligned} 3\ddot{x} + l\ddot{\varphi}_1 + l\ddot{\varphi}_2 &= 0 \\ l\ddot{\varphi}_1 + \ddot{x} + g\varphi_1 &= 0 \\ l\ddot{\varphi}_2 + \ddot{x} + g\varphi_2 &= 0 \end{aligned}$$

Question 3

Consider the system from question 2.

a) Derive the secular equation for the system.

b) Solve the secular equation and find the normal frequencies of the system.

c) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.

Question 4

Consider two satellites of the same mass m. The first satellite can be considered as a sphere of radius r and moment of inertia $I = \frac{2}{3}mr^2$. The second satellite can be considered as a uniform ball of the same radius r and moment of inertia $I = \frac{2}{5}mr^2$. Each satellite is rotating around its center. Angular velocities are of equal magnitude ω. They dock together, the docking is instant. Determine the rotation of the "space station" that is formed as a result of this docking in the following cases.

a) Their initial angular velocities are parallel and they dock along the axis parallel to the angular velocity, see Fig. 3

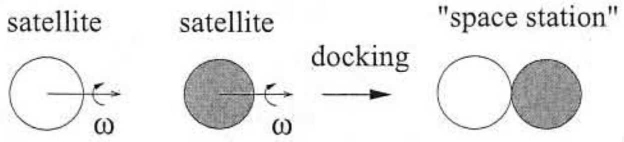

FIG. 3:

b) Their initial angular velocities are parallel and they dock along the axis perpendicular to the angular velocity, see Fig. 4

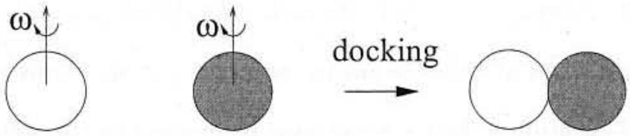

FIG. 4:

c) Their initial angular velocities are perpendicular and they dock along the axis parallel to one of the angular velocities, see Fig. 5

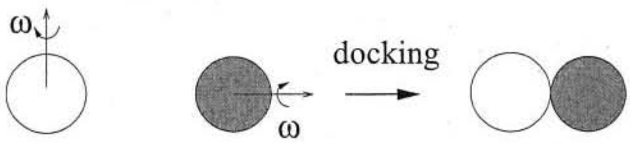

FIG. 5: