THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2008

PHYS2010

MECHANICS

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

Answer each question in a separate book

All questions ARE of equal value

Candidates may not bring their own calculators.

The following materials will be provided by the Enrolment and Assessment Section: Calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Candidates may keep this paper.

PPS

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{b}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$U_{eff}(r) = \frac{M^2}{2mr^2} + U(r)$$
$$\varphi = \pm\int \frac{\left(M/r^2\right)dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

Lagrangian

$$L = T - U$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Question 1

A particle of mass m moves under the influence of an isotropic harmonic restoring force on a two-dimensional plane.

$$\mathbf{F} = -k\mathbf{r}$$

a) Use Newton's laws of motion to determine the equation of motion for this system.

b) Show that this equation of motion is separable into two equations of motion.

c) Solve the two equations of motion as a function of time.

d) Derive the path equation for the motion of the particle.

e) Describe the types of path that will be followed by the particle under the influence of the isotropic harmonic restoring force.

The above restoring force is replaced by a non-isotropic harmonic force:

$$\mathbf{F} = -k_1 x\mathbf{i} - k_2 y\mathbf{j}$$

where the two spring constants k_1 and k_2 are not equal.

f) Describe the motion of a particle moving under the influence of this non-isotropic force.

g) Sketch the general orbit for arbitrary spring constants.

Question 2

A particle moves in the attractive central potential

$$U(r) = -\frac{\alpha}{r^n}.$$

Angular momentum of the particle is nonzero, $M \neq 0$. Consider cases $n = 2$ and $n > 2$. The motion starts at a finite distance r_0 and initially the radial velocity is directed towards the centre.

a) (2 marks) Sketch all possible trajectories dependent on parameters. Explain how the parameters influence the shape of the trajectory.

Below, consider cases where the particle falls to the centre for both $n = 2$ and $n > 2$.

b) (4 marks) Is the time necessary to reach the origin, $r = 0$, finite or infinite? Explain your answer.

c) (4 marks) Is the number of revolutions before the particle gets to the origin finite or infinite? Explain your answer.

Question 3

A double pendulum consists of two masses m_1 and m_2 in the gravitational field g directed down. The mass m_1 is attached to a massless rod of length l_1 which hangs from a pivot attached to the ceiling, while m_2 is attached to another massless rod of length l_2 connected by a pivot with m_1.

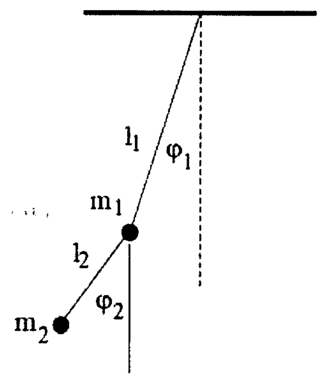

FIG. 1:

The dashed lines just show the vertical direction. The masses can move only in the plane of Figure, so the natural dynamical variables are angles φ_1 and φ_2.

a) (5 marks) Derive Lagrangian of the pendulum in terms of φ_1 and φ_2.

b) (5 marks) Derive equations of motion of the pendulum. Do not assume that the angles are small.

Question 4

Two equal masses m can move without friction. The masses are connected by a spring with elastic constant k. An identical spring connects the left mass to the wall.

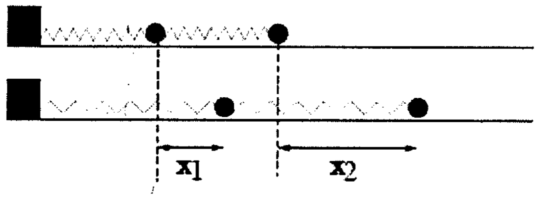

FIG. 2:

The masses can move only along one line as it is shown in Figure. The top part of the figure shows the equilibrium position (springs are un-stretched). The lower part shows a deviation from the equilibrium. So the coordinates x_1 and x_2 denote displacements of each mass from the corresponding equilibrium position.

a) (2 marks) Using the second Newton's law (or otherwise) write down equations of motion in terms x_1 and x_2.

b) (2 marks) Derive the secular equation for the system.

c) (3 marks) Solve the secular equation and find the normal frequencies of the system.

d) (3 marks) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.