THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE/JULY 2007

PHYS2010

Mechanics

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions ARE of equal value

Candidates may not bring their own calculators.

The following materials will be provided by the Enrolment and Assesment Section: Calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

PPS

The formula sheet.

Central field

$$U_{eff}(r) = \frac{M^2}{2mr^2} + U(r)$$

$$\varphi = \pm \int \frac{\frac{M}{r^2}\, dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

$$t = \pm m \int \frac{dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

Lagrangian

$$\mathcal{L} = T - U\ .$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Symmetric top

$$\Omega_3 = \frac{M_3}{I_3}$$

$$\Omega_{pr} = \frac{M}{I_1}$$

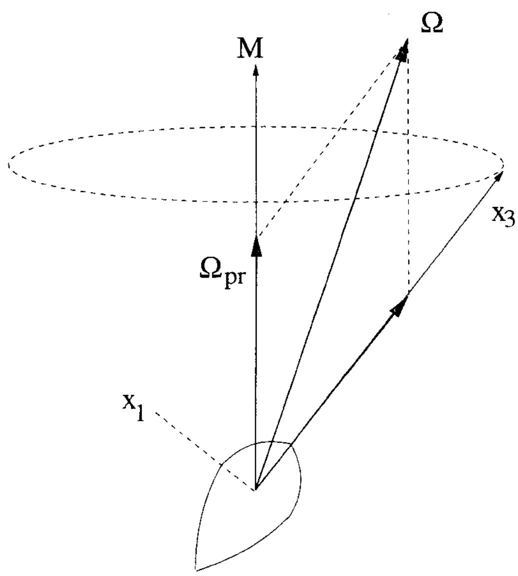

FIG. 1:

Question 1

A particle moves in the attractive central potential

$$U(r) = -Ve^{-\kappa^2 r^2} ,$$

where κ is real.

a) (5 marks) Sketch the effective potential. Clearly show different possible shapes of the potential.

b) (5 marks) At which values of angular momentum M can the particle move in the potential without escaping to infinity (finite motion)?
Hint: Your answer to question **a)** *can be helpful.*

Question 2

Two equal masses m can move without friction. The masses are connected by a spring with elastic constant k. An identical spring connects the left mass to the wall. The masses can move only along one line as it is shown

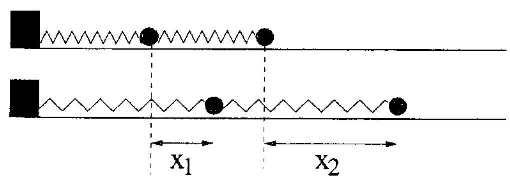

FIG. 2:

in Figure. The top part of the figure shows the equlibrium position (sprins are unstrached). The lower part shows a deviation from the equlibrium. So the coordinates x_1 and x_2 denote displacements of each mass from the corresponding equilibrium position.

a) (5 marks) Derive Lagrangian of the system in terms of x_1, x_2 and their time derivatives.

b) (5 marks) Using the Lagrangian derive equations of motion of the system.

Question 3

Consider the system from question **2**.

a) **(2 marks)** Write down equations of motion in terms x_1 and x_2. You can use the Lagrangian derived in question **2**, or alternatively you can start straight from the second Newton's law.

b) **(2 marks)** Derive the secular equation for the system.

c) **(3 marks)** Solve the secular equation and find the normal frequencies of the system.

d) **(3 marks)** Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.

Question 4

Consider two identical satellites. Each satellite can be considered as a uniform ball of radius r, mass m and moment of inertia $I = \frac{2}{5}mr^2$. Each satellite is rotating around its center. Angular velocities are of equal magnitude ω. They dock together, the docking is instant. Determine the rotation of the "space station" that is formed as a result of this docking in the following cases.

a) **(2 marks)** Their initial angular velocities are parallel and they dock along the axis parallel to the angular velocity, see Fig. 3

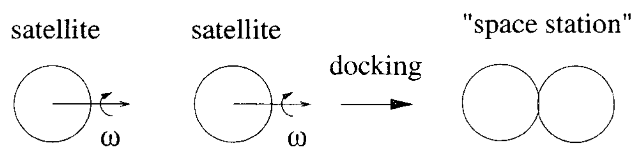

FIG. 3:

b) **(3 marks)** Their initial angular velocities are parallel and they dock along the axis perpendicular to the angular velocity, see Fig. 4

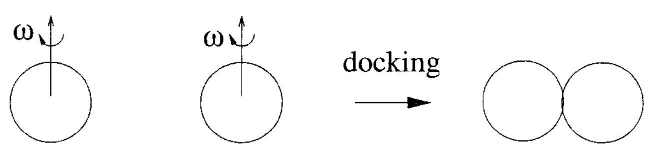

FIG. 4:

c) **(5 marks)** Their initial angular velocities are perpendicular and they dock along the axis parallel to one of the angular velocities, see Fig. 5

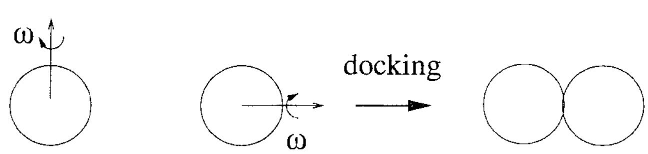

FIG. 5: