THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION TEST – APRIL 2005

PHYS2010 – MECHANICS

Time allowed – 50 minutes

Total number of questions – 3

Answer ALL questions.

The questions are of not of equal value.

This paper may be retained by the candidate.

NO calculators are to be used for this paper.

All answers must be in ink.

Except whre they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{b}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$U_{eff}(r) = \frac{M^2}{2mr^2} + U(r)$$
$$\varphi = \pm\int \frac{\left(M/r^2\right)dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

Lagrangian

$$L = T - U$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Question 1 (5 marks)

A force is given by:

$$\mathbf{F}(x,y,z) = \left(2xy + z^3\right)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$$

(a) Show that this force is conservative.
(b) Find the potential energy, V.
(c) Determine the work done in moving an object in this force field form (1,-2,1) to (3,1,4).

Question 2 (7 marks)

A simple harmonic oscillator is described by the equation:

$$m\ddot{x} + kx = 0$$

This oscillator is altered by the addition of a damping term:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(a) Give expressions for the resonant frequency of the original oscillator and the subsequent damped oscillator.
(b) How are these frequencies related to each other?

The damped oscillator is now driven by a harmonic driving force given by:

$$F = F_o \cos\omega t$$

(c) At what frequency will this system resonate?
(d) Describe the behaviour of the resonance as the damping constant is increased from zero to infinity.
(e) Describe how the phase of the oscillating particle motion is related to the driving force as a function of frequency
(f) How does this relationship depend on the degree of damping?

Question 3 (8 marks)

A particle moves in one dimension under the action of a conservative force.

(a) Show that the formal solution to the equation of motion is given by:

$$t - t_o = \int_{x_o}^{x} \frac{dx}{\sqrt{\left(\frac{2}{m}\right)\left(E - V(x)\right)}}$$

where the particle is at x_o at time t_o.

A stationary central force in three dimensions is given by:

$$\underline{\mathbf{F}} = f(r)\frac{\underline{\mathbf{r}}}{|\underline{\mathbf{r}}|}$$

(b) Show that this force is conservative.
(c) Show that the angular momentum of a particle moving in this force field is a constant of motion for this system.
(d) How does this restrict the motion of such a particle?
(e) Derive an expression for the magnitude of the angular momentum of such a particle in terms of plane polar coordinates and their time derivatives.