THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MID SESSION TEST - MAY 1998

PHYS2001 - MECHANICS AND COMPUTATIONAL PHYSICS

PHYS2999 - MECHANICS AND THERMAL PHYSICS

PHYS2991 - MECHANICS AND THERMAL PHYSICS

PAPER 1 - MECHANICS

Time allowed - 1 hour

Total number of questions - 3

Attempt **ALL** questions

The questions are **NOT** of equal value

This paper may be retained by the candidate

Candidates may not bring their own calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

QUESTION 1 **(5 marks)**

For the damped harmonic oscillator, the equation of motion can be written as:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(a) explain all terms in this equation.

The equation can be solved by finding the roots of the corresponding subsidiary equation:

$$m\alpha^2 + c\alpha + k = 0$$

This results in three distinct solutions that represent three types of physical behaviour: overdamping, critical damping and underdamping.

(b) Explain each of these modes of physical behaviour. Use sketches to illustrate the motion of the particle in each case.

Question 2 **(5 marks)**

A force is given by:

$$\mathbf{F}(x,y,z) = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$$

(a) Show that this force is conservative.

(b) Find the potential energy, V.

(c) Determine the work done in moving an object in this force field from (1,-2,1) to (3,1,4).

Question 3 (10 marks)

A central force is given by the equation:

$$\mathbf{f}(r) = -\frac{k}{r^3}\hat{\mathbf{r}}$$

where k is a constant and r is the distance from the centre. An object of mass m moves under the influence of this central force.

(a) Show that the potential producing this force is given by:

$$V(r) = -\frac{k}{2r^2}$$

(b) Derive an expression for the effective potential, U(r), of the particle, mass m, moving in this central field.

The form of the equation for the effective potential produces three distinct curves that depend on the values of the parameters: m, the particle mass, h (or l) the angular momentum per unit mass and k the force constant.

(c) Sketch the three possible effective potential curves as functions of radius. Label each curve with the conditions on m, h (or l) and k that are appropriate to the curve.

(d) What type of motion will be experienced by the particle in each of the three cases?

(e) Use Newton's 2nd law to show that the equation of motion for the particle reduces to:

$$\ddot{r} + \frac{1}{r^3}\left(\frac{k}{m} - h^2\right) = 0$$

(f) Hence, show that the orbit equation for the particle is given by:

$$\frac{d^2u}{d\theta^2} - u\left(\frac{k}{mh^2} - 1\right) = 0$$

where $u = \frac{1}{r}$ and θ is the angle in plane polar coordinates.

This is the equation of an undamped harmonic oscillator in terms of u and θ. Hence it has a general solution:

$$u(\theta) = A_+ e^{+i\theta\sqrt{1-\frac{k}{mh^2}}} + A_- e^{-i\theta\sqrt{1-\frac{k}{mh^2}}}$$

(g) Use this general solution to derive the equation of the orbit, r(θ), in terms of the normal plane polar coordinates, r and θ, for the three cases that depended on the relationship between k, h (or l) and m (as per (c)).

(h) Describe the motion and orbit in each case.

(i) In a real case, k and m are fixed by the nature of the force field and the type of particle interacting with it. Hence, the value of the angular momentum per unit mass, h (or l) determines the type of orbit. Discuss the physical significance of the transition between orbit types produced by variations in h (or l).