Question 1 [16 marks]

A) Two capacitors, $C_1 = C$ and $C_2 = 3C$, are each charged to a voltage V by an external power supply. Once fully charged the power supply is disconnected from each capacitor.

(a) What is the resultant energy stored on the two capacitors when they are fully charged? [2 marks]

Now the two capacitors are connected together, with positive plate to negative plate and negative plate to positive plate. Find, once equilibrium has been reached:

(b) The charge on each capacitor. [2 marks]

(c) The potential difference across each capacitor. [2 marks]

(d) The total energy stored on these capacitors now [2 marks]

(e) Account for the differences in your answers to (a) and (d). [2 marks]

B) A parallel plate capacitor is formed by two square plates, each of side length *a*. The separation of the plates is *d*. However there was a deviation during manufacture, causing the plates to be misaligned. The two square plates are offset from being parallel by a small angle θ, as shown in Figure Q1.

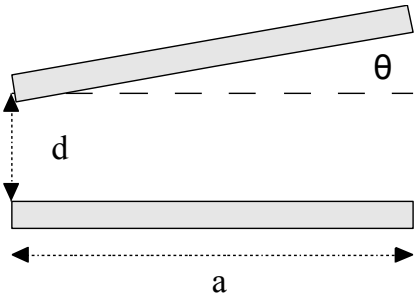

Figure Q1

Show that the capacitance of this device can now be approximated as [6 marks]

$$C = \frac{\varepsilon_0 a^2}{d}\left[1 - \frac{a\theta}{2d}\right]$$

Question 2 [14 marks]

A) The diagram below, Figure Q2a, shows a conducting rod of length *L* being pulled along a horizontal, frictionless, conducting rails at a constant velocity **v**. A uniform vertical magnetic field **B** fills the region in which the rod moves. Assume that *L* = 10.8cm, *v* = 4.86m/s, and *B* = 1.18T. You may assume that the resistance of the rod is 415mΩ and that the resistance of the rails is negligibly small.

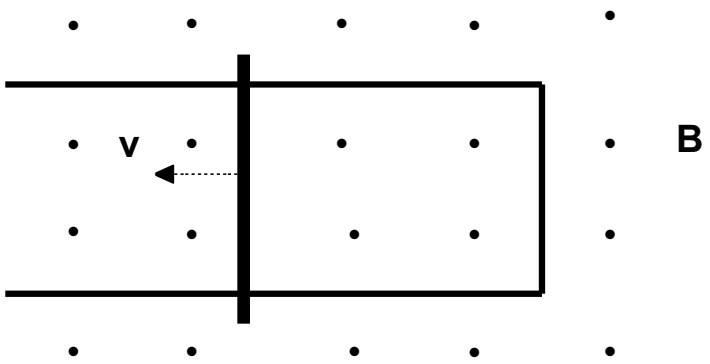

Figure Q2a

(a) Determine the magnitude and direction of the current that is induced in the loop. [4 marks]

(b) At what rate is electrical energy being dissipated in the rod? [2 marks]

(c) Find the force that must be applied by an external agent to the rod to maintain its motion. [2 marks]

(d) At what rate does this external force do work on the rod? Compare this answer with the answer to (b). [2 marks]

B) Charged particles of mass 238u and charge -2e are to be used in a mass spectrometer. These particles are first accelerated to a speed of $5.4\times10^{6}\ \mathrm{ms}^{-1}$. These particles then pass into a region of magnetic field of strength *B* = 520mT, directed as shown in Figure Q2b.

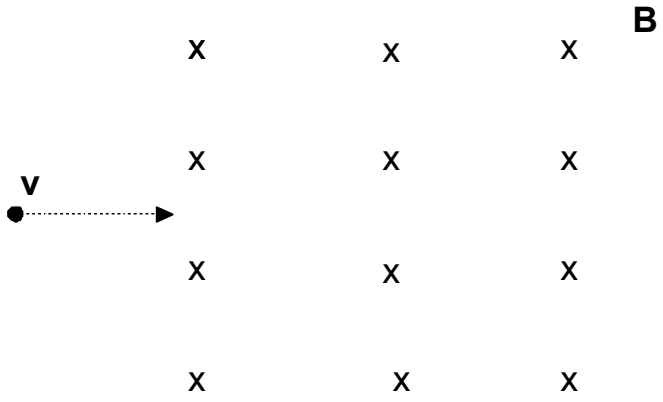

Figure Q2b

Determine the magnitude and direction of an externally applied electric field such that there would be no net force on these particles. [4 marks]

Question 3 [8 marks]

A light ray of wavelength λ = 557nm, initially in air, strikes an 78° prism at P, as shown in Figure Q3. This ray is refracted there at P and at Q to such an extent that it just grazes the right-hand prism surface at Q. The refractive index of the glass is *n* = 1.58.

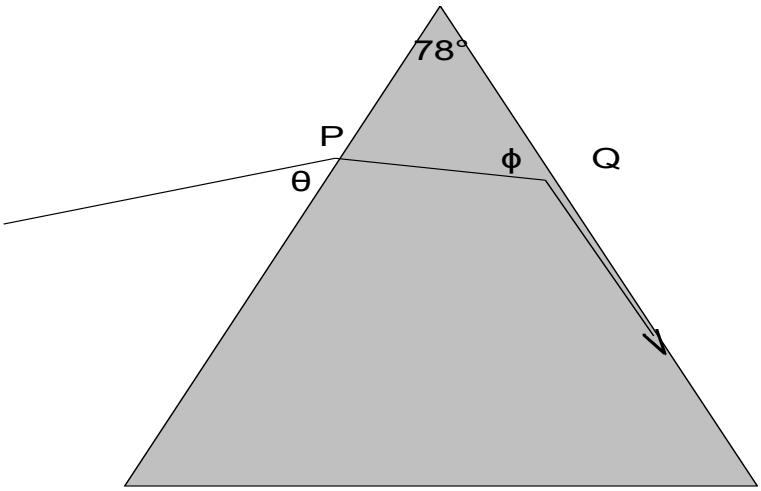

Figure Q3

(a) Determine the value of the angle ϕ at Q. [3 marks]

(b) Determine the grazing angle of incidence to the prism at P, θ. [3 marks]

(c) Show, by ray diagrams, what happens if the grazing angle of incidence at P is slightly greater and slightly less than θ. [2 marks]

Question 4 [12 marks]

A) Laser light of wavelength λ is incident onto a pair of narrow slits of width *a* and spacing *d*, as shown in Figure Q4.

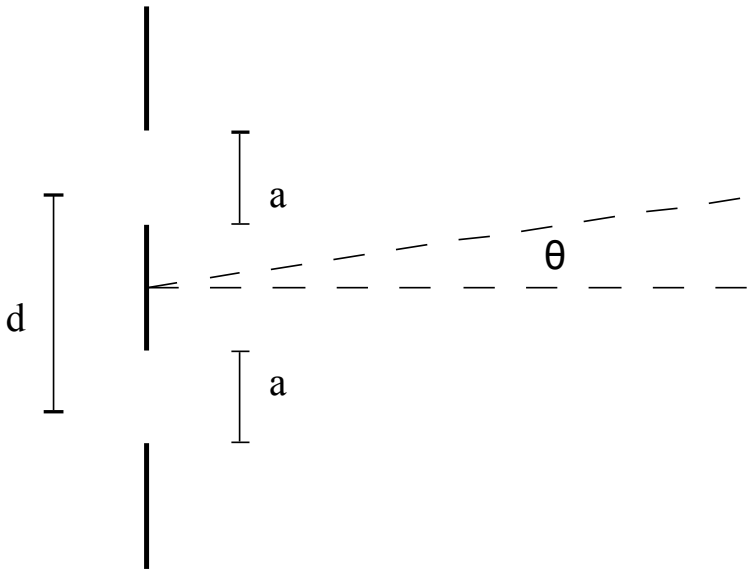

Figure Q4

Consider the combined interference and diffraction pattern visible in the far-field.

(a) Write down an expression for the intensity as a function of angle of the angle measured from the central axis, θ, in the far-field. Explain carefully where each term comes from. [4 marks]

(b) If $d = 2a$, how many interference fringes lie within the central diffraction envelope? [2 marks]

(c) If we put $d = a$ the two slits coalesce into a single slit of width $2a$. Show that your expression in (a) reduces to the diffraction pattern for such a slit in this case. [2 marks]

B) A sheet of glass having a refractive index of 1.40 is to be coated with a film of material having an index of refraction of 1.55 such that green light (wavelength 525nm) is preferentially transmitted.

What is the minimum thickness of the film that will achieve the result? [4 marks]

Question 5 [7 marks]

Satellites and spacecraft in orbit about the Earth can become charged due, in part, to the loss of electrons caused by the photoelectric effect induced by sunlight on the space vehicle's outer surface. Suppose that a satellite is coated with platinum, whose work function is $\phi = 5.32\text{eV}$.

(a) Find the smallest frequency photon that can eject a photoelectron from platinum. [3 marks]

(b) In order to minimise this effect on a satellite, is it better to choose a metal with a larger or smaller work function? Explain your answer. [2 marks]

(c) Would this induced charge induce an Electric field inside the satellite? Why/why not? [2 marks]

Question 6 [8 marks]

Consider an electron trapped in an infinite, one-dimensional well of width 98.5pm. Suppose it is in the $n = 15$ energy state.

(a) What is its energy? Express your answer in eV. [4 marks]

(b) What is the uncertainty in its momentum? [2 marks]

(c) What is the uncertainty in its position? [2 marks]

Question 7 [10 marks]

An electron in the $(n,l,m_l) = (1,0,0)$ state of the Hydrogen atom has the radial wave-function

$$\Psi_{100}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where $a_0 = 0.529\text{nm}$ is the Bohr radius.

(a) Explain what each of the quantum numbers, (n,l,m_l), represent. [3 marks]

(b) Explain why, referring to the physical meaning of the wave-function, the probability of finding the electron in a region between two spherical shells of radii r and $r + dr$ is: [3 marks]

$$P_r(r)dr = \Psi^2(r)4\pi r^2\, dr$$

(c) Hence, or otherwise, estimate the probability of finding the electron within 1.5 Bohr radii of the nucleus. [4 marks]