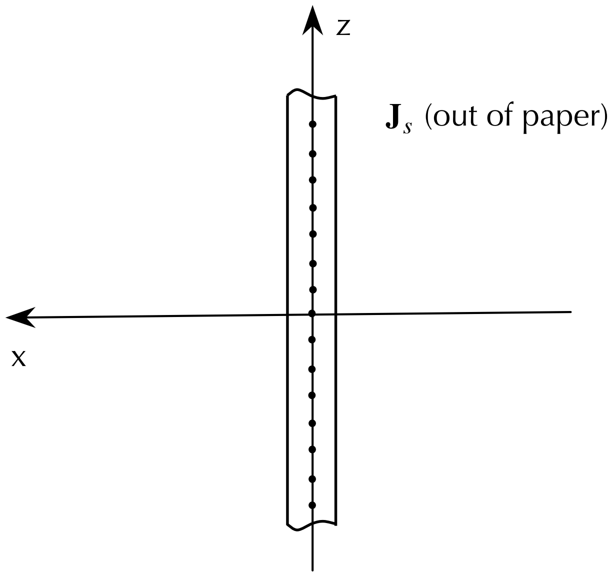
Question 1. [20 Marks]

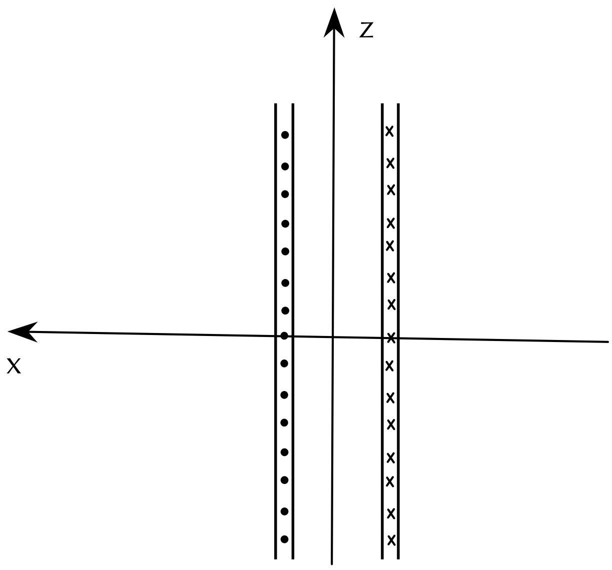


Consider an infinite conducting sheet lying in the yz-plane, as shown in the Figure (the y-axis comes out of the plane of the paper). Suppose the sheet is carrying a uniform current in the y direction, out of the plane of the paper. Let J_s be the linear current density, i.e., the current per unit length measured along the z axis. The dots in the figure show the current direction (i.e., out of the plane of the paper).

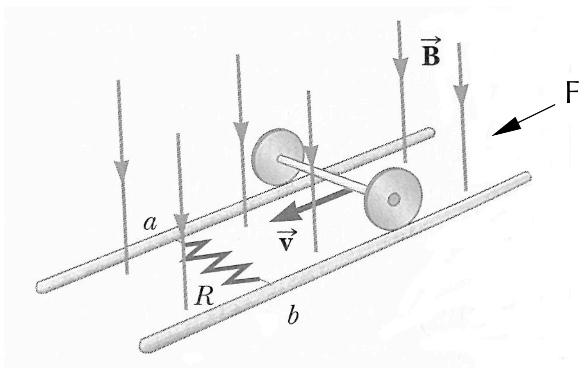
[a] How does the direction and relative strength of the magnetic field vary at all points in space due to the current flowing in the sheet? You should give reasons for your answer, and sketch the field. [Hint: consider the magnetic field from a single charge moving with the current, and use symmetry.]

[b] By drawing an appropriate amperian loop, calculate the magnitude of the magnetic field, B, as a function of position and J_s.

[c] Suppose we place a second infinite conducting sheet parallel to the first one, and carrying the same current density as the first sheet, but in the -y direction, as shown in the Figure below, where the dots and crosses indicate the direction of current flow. Derive and sketch the magnitude and direction of the magnetic field throughout space.



Question 2. [20 Marks]



The Figure shows a rolling axle of length 1.50m travelling at a constant speed v=5.00m/s along rails. A 0.200 ohm resistor R is placed between points a and b as shown. You can assume that the rails, wheels and axle are conductors with zero resistance and make good electrical contact with each other. A uniform magnetic field, B, of strength 5.0×10^{-4} T is directed vertically downwards.

[a] Calculate the induced current I in the resistor.

[b] What is the direction of current flow in the resistor? Give reasons for your answer.

[c] Calculate the force F that is required to maintain the velocity of the axle.

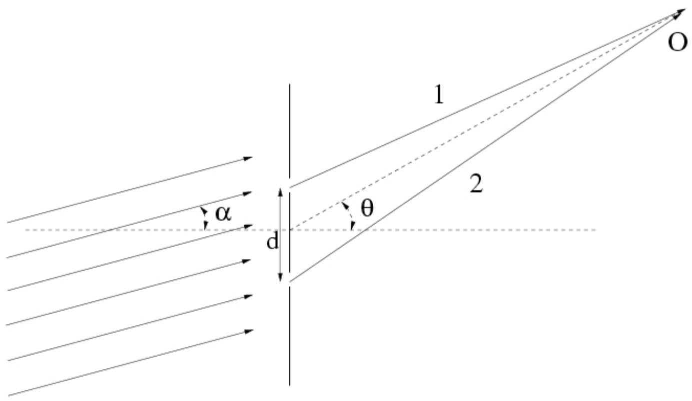
[d] The force F is doing work, but since the velocity of the axle is constant, there is no increase in the kinetic energy of the axle. So, how is energy conserved?

[e] Does the direction of current flow through the resistor change once the axle has rolled past the position of the resistor? Give reasons for your answer.

Question 3. [20 Marks]

A coherent beam of light with wavelength $\lambda = 600$ nm is incident from the left on a double slit arrangement. The beam makes an angle $\alpha = 20^{\circ}$ with the axis, as shown in the Figure below.

The width of each slit is small compared to the wavelength. There is interference between rays 1 and 2 that originate from the slits. The observation point O is very far, so the rays 1 and 2 are practically parallel and make an angle θ with the axis, as shown in the Figure. The distance between the slits is d = 2000 nm.



(a) Derive an equation that determines the values of θ that correspond to interference maximums.

(b) Derive an equation that determines the values of θ that correspond to interference minimums.

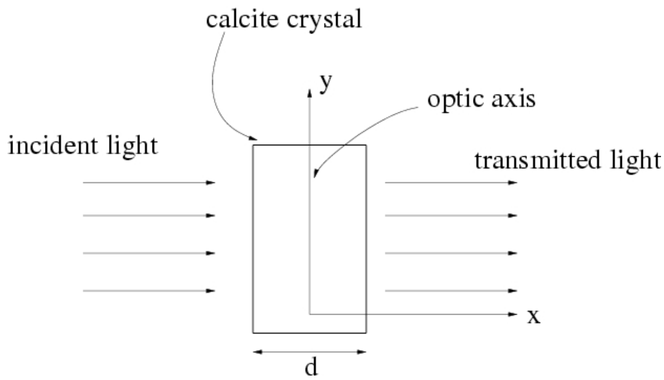
(c) Solve the equation derived in part (**a**) and hence find the positions of *all* the interference maximums.

(d) Derive a formula for the light intensity at the observation point O as a function of the angle θ.

Question 4. [20 Marks]

A calcite crystal is birefringent. The index of refraction for ordinary rays is n_0=1.658 and the index of refraction for extraordinary rays is n_e = 1.486.

You will recall that the ray with polarization perpendicular to the optic axis of the crystal is called "ordinary" and the ray with polarization parallel to the axis is called "extraordinary".
A coherent light beam with wavelength λ =600 nm is incident from the left on a calcite crystal of thickness *d*. The light is moving along the x-axis, as is shown in the Figure.
The optic axis of the calcite crystal is directed along the y-axis, see the Figure. The z-axis is perpendicular to the plane of the picture.



The incident light is linearly polarized with electric field given by the following formula

$$\vec{E} = E_0\left\{\vec{j}\cos(kx - \omega t) + \vec{k}\cos(kx - \omega t)\right\}.$$

Here $\vec{j}$ and $\vec{k}$ are, as usual, the unit vectors along the y and z axes. The wave number is $k = 2\pi/\lambda$.

(a) Sketch how the electric field of the incident wave oscillates with time in the yz-plane at a given value of x.

(b) Find the minimum value of the crystal thickness *d* that is sufficient to transform the linearly polarized incident light to circularly polarized transmitted light.
The electric field in the circularly polarized wave is

$$\vec{E} = E_0\left\{\vec{j}\cos(kx - \omega t + \alpha) - \vec{k}\sin(kx - \omega t + \alpha)\right\}$$

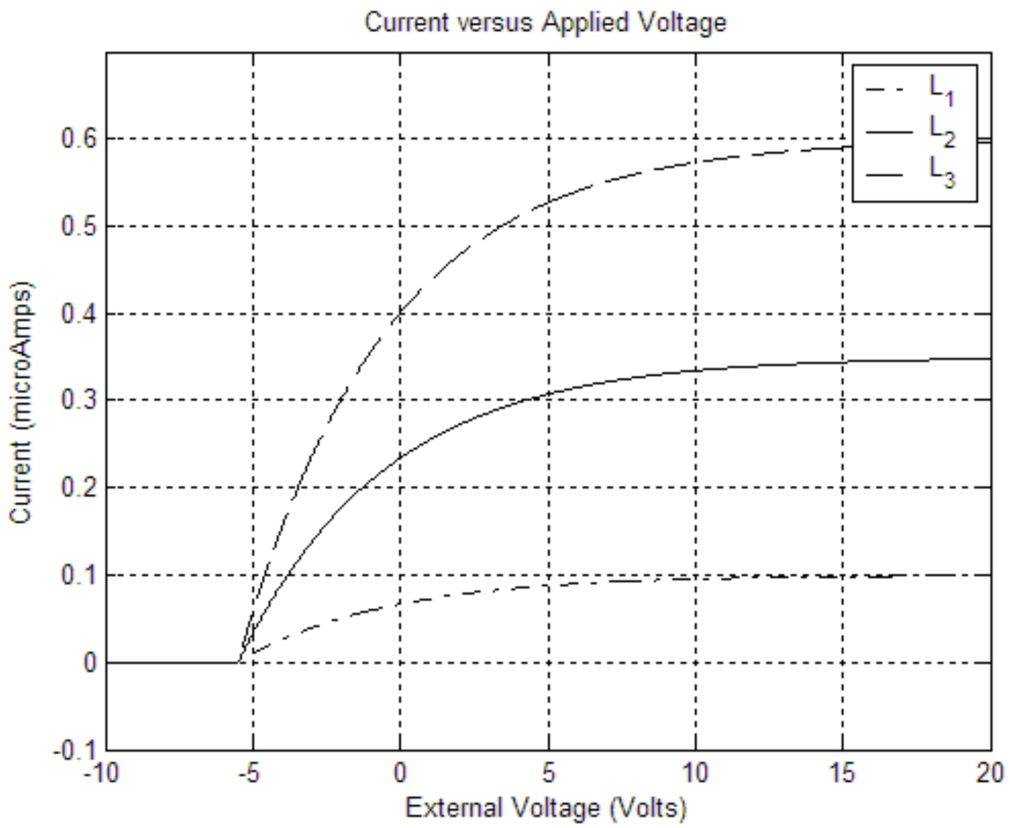
where α is some phase shift.

(c) Sketch how the electric field in the circularly polarized wave oscillates with time in the yz-plane at a given value of x.

(d) What will be the polarization of the transmitted light if the crystal thickness is tripled compared to that found in the point **b**?
Sketch how the electric field of the transmitted wave oscillates with time in the yz-plane in this case.

Question 5. [12 marks]

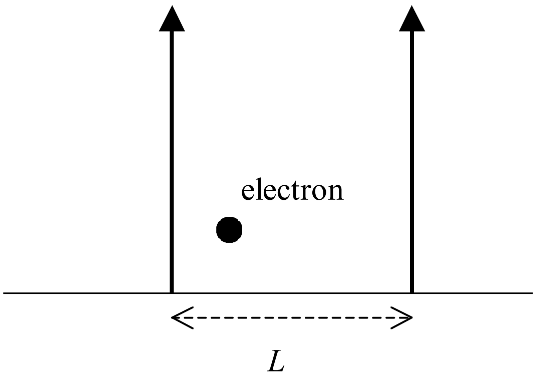
Light of wavelength λ = 320 nm is shone on a metal electrode in an evacuated tube. A voltage can be applied between this electrode and another electrode, and the resultant current around the circuit is measured with an ammeter. The current as a function of the voltage applied is shown in the graph below for three different light intensities,
$L_1 < L_2 < L_3$.



a) Draw a diagram of the experimental set-up, clearly indicating the polarity of the applied voltage to match the graph as shown.
b) Explain one observation of this graph that cannot be accounted for by the classical wave theory of light.
c) Describe ONE OTHER characteristic of this experiment that could not be explained by classical theory.
d) The intersection of these graphs with the voltage axis is at –5.5 V. Calculate the threshold frequency of this metal.
e) If the light had an intensity of 5 $\mathrm{mW/m^2}$ uniformly illuminating the metal electrode of area 10 $\mathrm{mm^2}$, calculate the saturation current that would result. Assume the process is 80% efficient (that is, 80% of incident photons eject an electron).

Question 6. [10 marks]

Imagining the world to be one-dimensional, an electron is confined to an infinite potential well as shown in the diagram.



a) According to classical physics, what are the allowed energies of the electron in this well?
b) State de Broglie's hypothesis, and explain the meaning of all symbols in any equations you write down.
c) Show that Quantum mechanics predicts that the energies of the electron in this box are:
$$E_n = \frac{n^2 h^2}{8 m_e L^2}, \quad n = 1, 2, 3, \ldots$$
d) Derive a formula for the wavelength of a photon emitted when the electron drops from the $n = 4$ level to the $n = 2$ level.

Question 7. [10 marks]

a) Explain the meaning of the following equation:
$$\Delta x \cdot \Delta p \geq \hbar$$
b) Describe how the Heisenberg Uncertainty principle is a natural consequence of the wave-particle duality of nature.
c) When transitioning between two excited states, a certain atom emits a photon of wavelength $\lambda = 122.3$ nm. The two excited states have lifetimes of 12 ns and 23 ns respectively. Use the uncertainty principle to estimate the line-width, $\Delta\lambda$, of this spectral line. [Hint: $\Delta E \cdot \Delta t \approx \hbar$].

Question 8. [8 marks]

Copper is a monovalent atom of atomic mass 63.54 amu and density 8.96 $\mathrm{g/cm^3}$.

a) Estimate the number density of conduction electrons in copper at 0 K. [Hint: Each Copper atom contributes one electron to the conduction band.]
b) A wire of Copper of radius 2 mm carries a current of 1.3 A at room temperature. Estimate the drift velocity of electrons in this Copper wire.
c) Compare this drift velocity to the approximate thermal velocity of these electrons, at room temperature.